Level 1 Calculus Final Exam
2013

Day 1 – 50 minutes

Name: _____ Block: _____

Circle Teacher Name LeBlanc Normile

Instructions

Write answers in the space provided and ***show all work***.
Calculators okay but observe instructions on individual problems.
For any decimal answers, give the answer to three decimal places.
Partial credit will not be given if work is not shown.

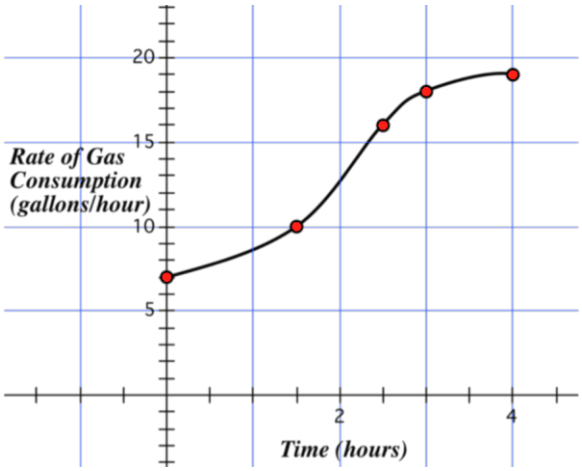
Question	Max Points	Score
1	4	
2	4	
3	6	
4	5	
5	4	
6	7	
TOTAL:	**30**	

1. Find the particular (specific) solution of the differential equation. Solve for y. Simplify your answer.

$\frac{dy}{dx} = \frac{2x}{y^2}$ and goes through the point (1, 0)

2. The rate of gasoline consumption (in gallons per hour) for a NASCAR racecar recorded during a race is given by a continuous and strictly increasing function C of time t. The graph of C and a table of selected values of $C(t)$, for the time interval $0 < t < 4$ hours, are shown below.

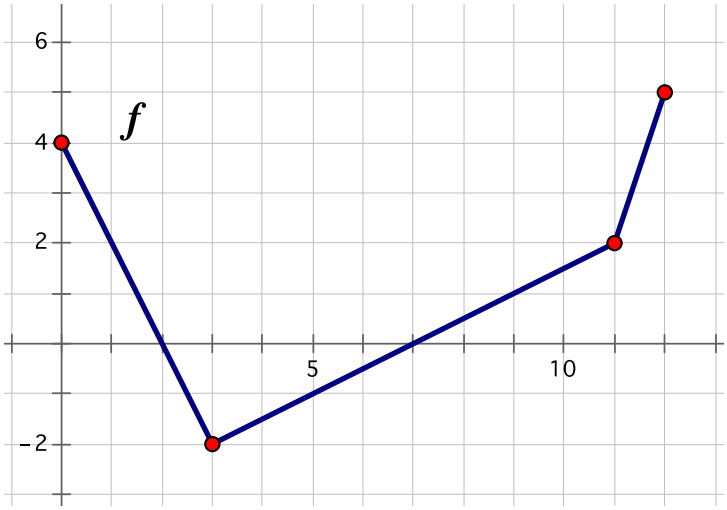
t (in hours)	***C(t)*** (in gallons/hour)
0	7
1.5	10
2.5	16
3	18
4	19



a. Approximate the value of $\int_0^4 C(t)\,dt$ using a **Right-handed Riemann Sum** (rectangular approximation method) with the 4 subintervals indicated by the data in the table. Show all your work.

b. Explain the meaning of your answer in part a in terms of the racecar. Include appropriate units.

3. The graph of f (which is made up of line segments) is shown below. Let $g(x) = \int_0^x f(t)\,dt$.



Use the graph above to evaluate each expression below.

a. *g(0)=* _____

b. *g(7)=* _____

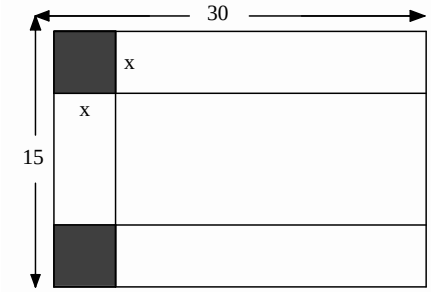
c. *g'(5)=* _____

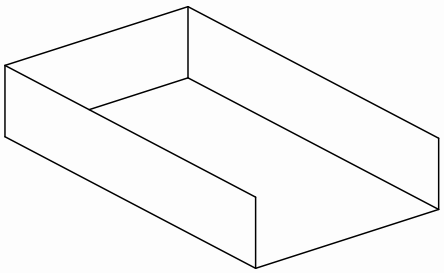
d. *g''(8)=* _____

e. At what x value(s) does g have a relative minimum? _____

f. At what x value(s) does g have an inflection point? _____

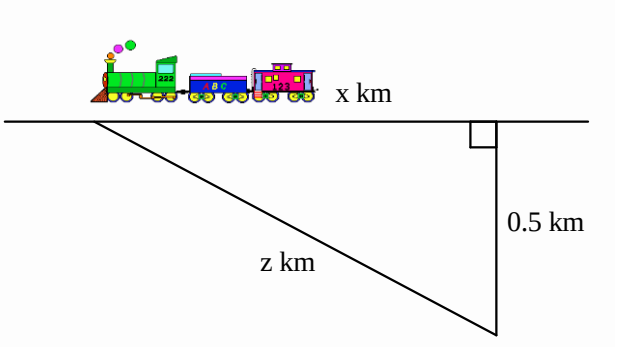
4. A box with an open top and one open side is to be formed from a piece of cardboard 15” by 30” by cutting squares from two corners along one of the 15" sides as shown. Find the dimensions that maximize its volume analytically (using derivatives).



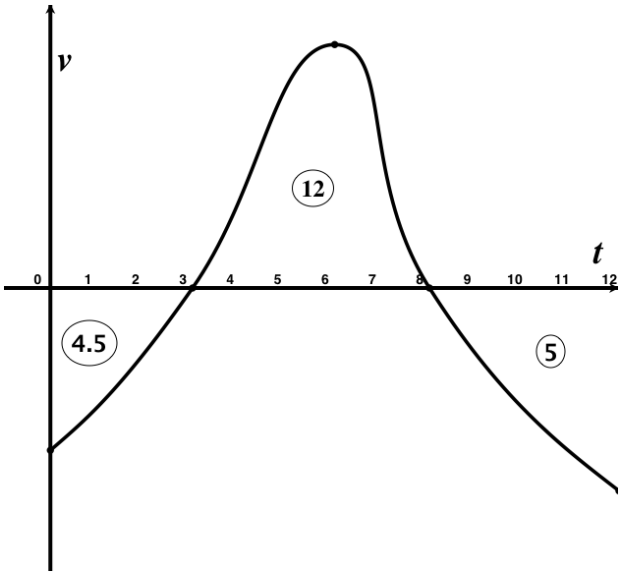


Height _____ Length _____ Width _____

5. A train is traveling a 0.8 km/min along a long straight track, moving in the direction shown. You are standing 0.5 km from the track. How fast is the distance from you changing when the train is 1 km away from you (z = 1km)?



6. An object's **velocity** function is given by the graph below. ***t*** is time in seconds, and ***v(t)*** is velocity in meters per second. $0 \le t \le 12$ The numbers in circles are areas between the graph and the *x*-axis. The object's initial position is $s(0) = -4m$.



Include the proper units in your answers to the following questions.

a) What is the object's **position** at time $t = 8$?

b) What is the object's **total distance traveled** between $t = 0$ and $t = 12$?

c) When is the object accelerating in the negative direction?

d) When does the object change direction? List each time and whether the change is positive-to-negative or negative-to-positive direction.

e) When is the object's position a global minimum?

Level 1 Calculus Final Exam
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Day 2 – 50 minutes

Name: _____ Block: _____

Circle Teacher Name LeBlanc Normile

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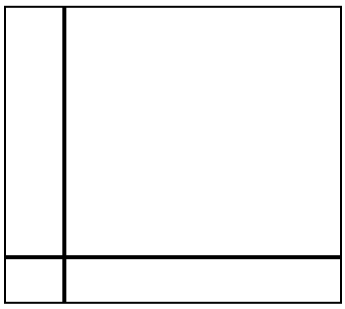
Question	Max Points	Score
7	11	
8	7	
9	7	
10	2	
11	2	
12	2	
13	5	
TOTAL:	**36**	

7. a. Sketch a graph of the region in quadrant I enclosed by the graph of $y = 2x^2$, the y-axis and the line $y = 8$.



b. Find the volume of the solid formed by rotating that region about the **y-axis**. (Calculate this integral analytically.)

c. Find the volume of the solid by rotating that region about the **x-axis.** (You can evaluate this integral on your calculator.)

8. Integrate each of the functions below analytically. Show your work.

a. $\int (e^x + 2\sin(x) + \sqrt{x})dx$

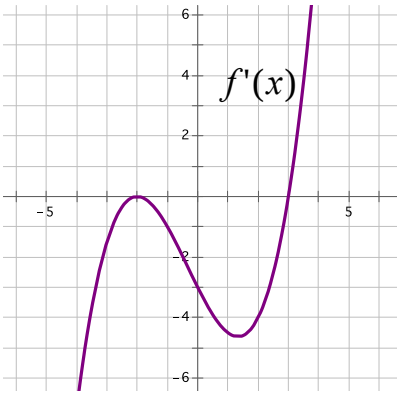
b. $\int (\cos(3x^2 + 1)2x dx$

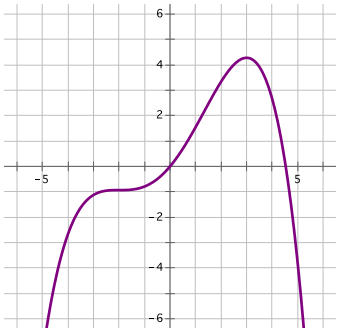
9. Evaluate each of the definite integrals analytically. Show your work.

a. $\int_1^3 \frac{x^2 + 4x + 2}{x} dx$

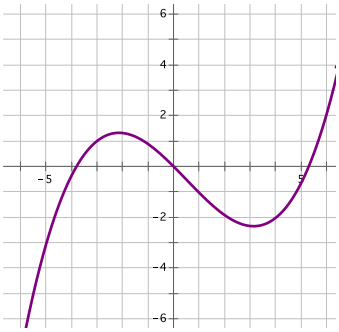
b. $\int_2^5 \frac{4x}{x^2 - 1} dx$

10. Given the graph of the **first derivative** ($f'(x)$), choose the graph that best represents the original function ($f(x)$).

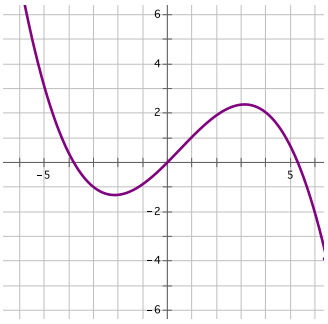




A



B



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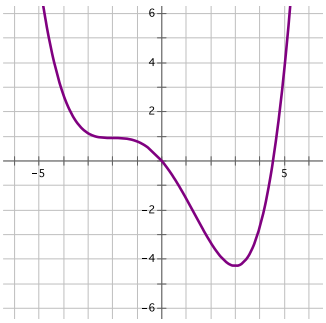
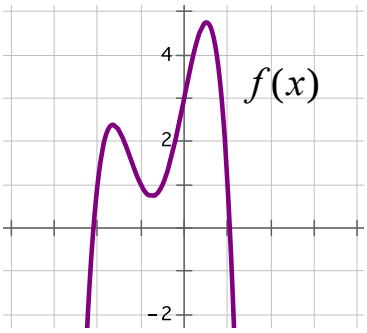
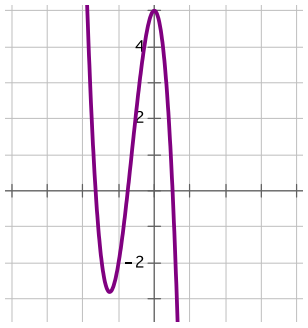


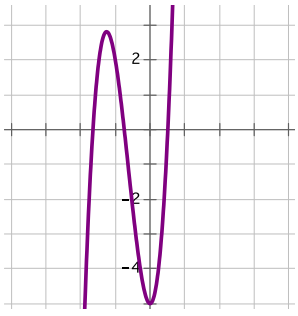
D

11. Given the graph of a function ($f(x)$), choose the graph that best represents the function's **second derivative** ($f''(x)$).





A



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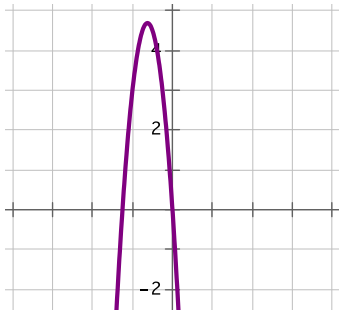

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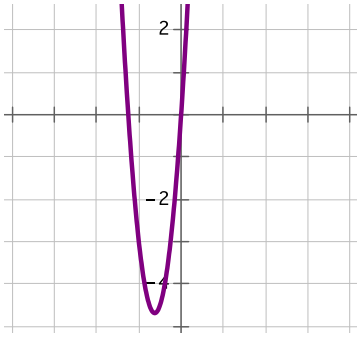
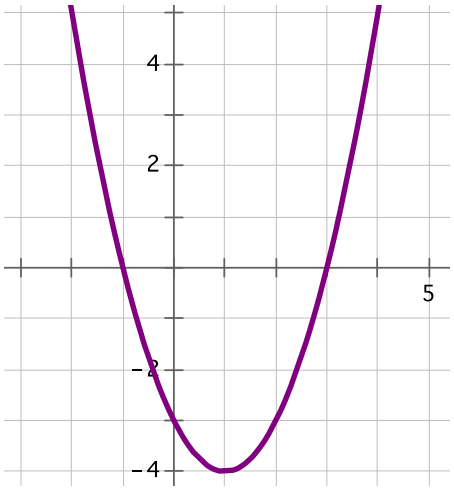


D

12. Given the graph of the **second derivative** ($f''(x)$) below, choose the description that best characterizes the original function ($f(x)$).



Graph of $f''(x)$

A. $f(x)$ has a minimum at x = -1 and a maximum at x = 3 and is decreasing on (-1, 3)

B. $f(x)$ has a minimum at x = -1 and a maximum at x = 3 and is increasing on (-1, 3)

C. $f(x)$ has points of inflection at x = -1 and x = 3 and is concave down on (-1, 3)

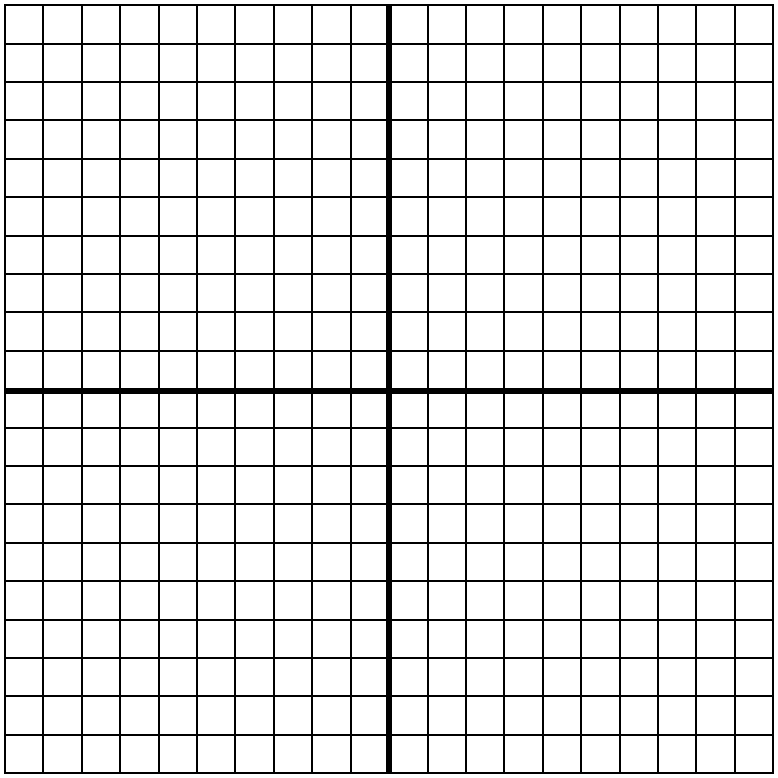
D. $f(x)$ has points of inflection at x = -1 and x = 3 and is concave up on (-1, 3)

E. $f(x)$ has a maximum at x = -1 and a minimum at x = 3 and is decreasing on (-1, 3)

F. $f(x)$ has a maximum at x = -1 and a minimum at x = 3 and is increasing on (-1, 3)

13.

a) Sketch a graph of the region enclosed by the graphs of $f(x) = x^2 + 2x + 1$ and $g(x) = 2x + 5$. Label the points of intersection.



b) Find the area of this region. Integrate **<u>analytically</u>** and show your work.