Algebra II, Level 1
Final Exam 2007

Lexington High School
Mathematics Department

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.
The exam has 2 parts. Directions for each part appear below.
The problems on this exam vary in point value between 2 and 8 points. Point values are specified next to each problem. In total, there are **176** points that you can earn. The course faculty will set a letter scale after the tests are graded.

Part A: No Calculator Section

55 points total
Carefully answer each question and show all work.
If your answer is incorrect, you may receive partial credit if you have shown some correct work.
A good pace would be to spend around 30 minutes on this part.

Part B: Calculator Section

121 points total
Carefully answer each question and show all work.
If your answer is incorrect, you may receive partial credit if you have shown some correct work.
A good pace would be to spend around 60 minutes on this part.

Directions

All students will start on Part A, the no-calculator part (colored blue). Once you turn it in, you may then take your calculator out and work on part B (colored white). **NO STUDENT WILL BE ALLOWED TO RETURN TO PART A AFTER PASSING IT IN.**

Read the directions for each question clearly. The directions provide guidance as to what form your answer needs to be in.

The Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Name _____

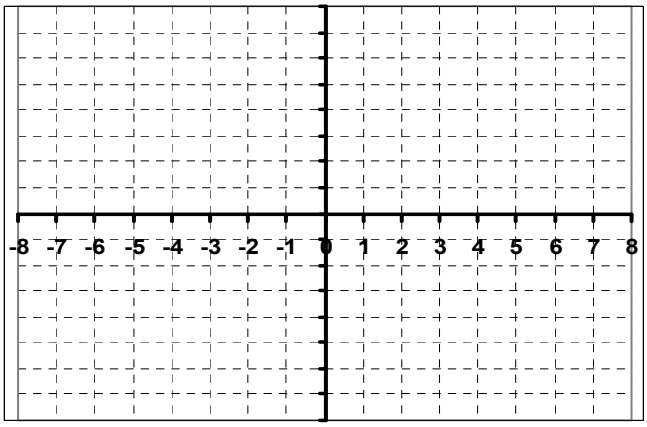
Teacher (circle): Engerman Haupt Nitsch Normile Verner

Class block (circle): A B C D E G H

PART A: NO CALCULATOR

1. **Sketch a graph of each function below.** Your graph should clearly show the shape of the function and correct placement of key features such as asymptotes, vertices, and intercepts. State the domain, range, and zero(s) in the space provided. Write the **equations** of the asymptotes, if any, in the space provided or on the graph.

a. $f(x) = -2x + 6$ [4 points]



Domain_____
Range _____
Zeros_____
Asymptotes (if any) _____

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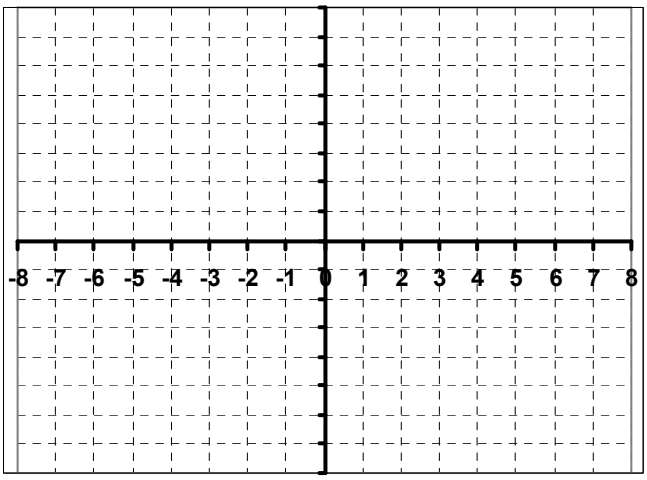
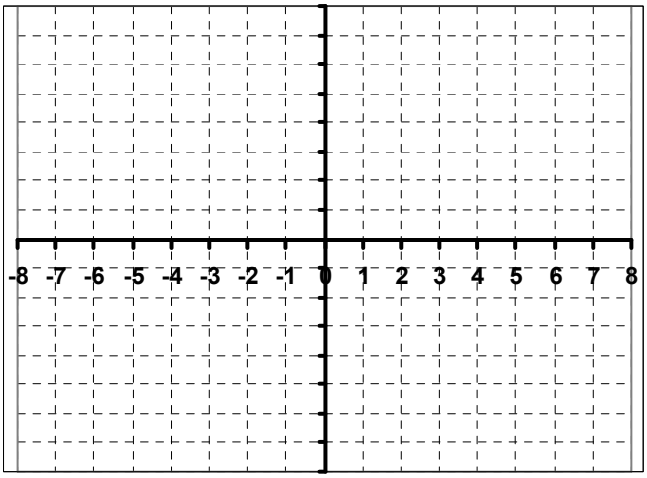
b. $f(x) = |x + 2| - 3$ [8 points]



Domain_____
Range _____
Zeros_____
Asymptotes (if any) _____

c. $f(x) = -\frac{1}{2} \cdot 2^x + 4$ [8 points]



Domain_____
Range _____
Zeros_____
Asymptotes (if any) _____

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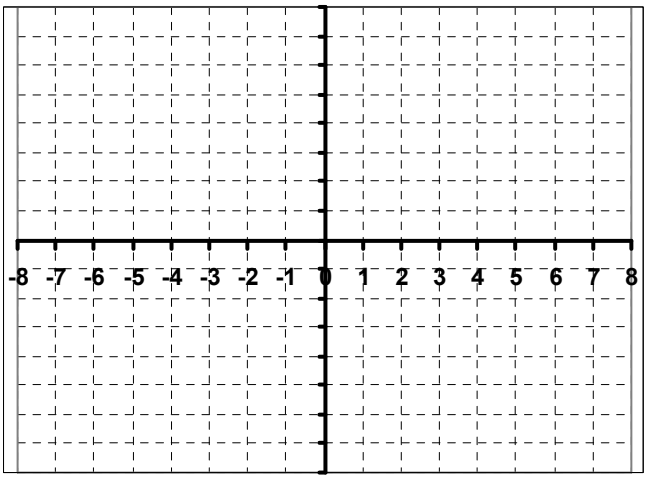
d. $f(x) = -(x+3)^2 + 4$ [8 points]



Domain_____
Range _____
Zeros_____
Asymptotes (if any) _____

2. Answer the following questions about the function $f(x) = -2x(x+3)(x-1)$. [6 points total].

a. What are the zeros of $f(x)$?

b. Which describes the end behavior of $f(x)$?

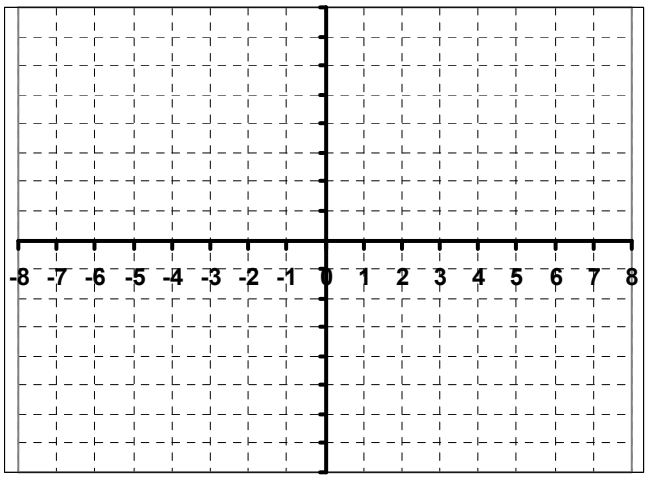
i) $as\ x \to \infty \quad f(x) \to \infty \quad and \quad as\, x \to -\infty \quad f(x) \to \infty$

ii) $as\ x \to \infty \quad f(x) \to \infty \quad and \quad as\, x \to -\infty \quad f(x) \to -\infty$

iii) $as\ x \to \infty \quad f(x) \to -\infty \quad and \quad as\, x \to -\infty \quad f(x) \to \infty$

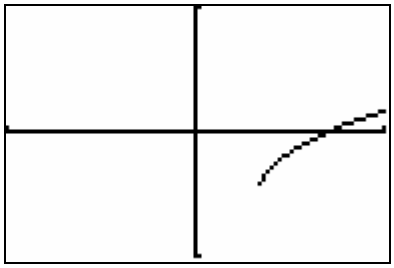
iv) $as\ x \to \infty \quad f(x) \to -\infty \quad and \quad as\, x \to -\infty \quad f(x) \to -\infty$

c. Sketch the graph of $f(x)$. Include the coordinates of the *x*-intercepts.



3. What is the domain of the function $f(x) = 2\sqrt{x-5} - 7$? [2 points]

4. Consider the following graph of the function $f(x)$. Which equation could be $f(x)$? [2 points]



a. $f(x) = \sqrt{x+3} - 2$

b. $f(x) = (x+3)^2 - 2$

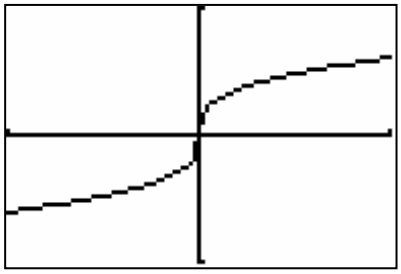
c. $f(x) = \sqrt[3]{x-3} - 2$

d. $f(x) = \frac{1}{2}\sqrt{x-3} - 2$

e. $f(x) = \frac{3}{x-3} - 2$

5. Consider the following graph of the function $g(x)$. Which equation could be $g(x)$? [2 points]

a. $g(x) = x^2$
b. $g(x) = x^3$
c. $g(x) = x^{-1}$
d. $g(x) = \sqrt[3]{x}$
e. $g(x) = \sqrt{x}$



6. Evaluate each of the following. [3 points each]

a. $\left(\frac{2}{5}\right)^{-2}$

b. $\left(\frac{1}{27}\right)^{2/3}$

c. $\log(1000)$

d. $\sqrt{18} - 4\sqrt{2}$

e. $2\log_2 3 - \log_2 36$

Name _____

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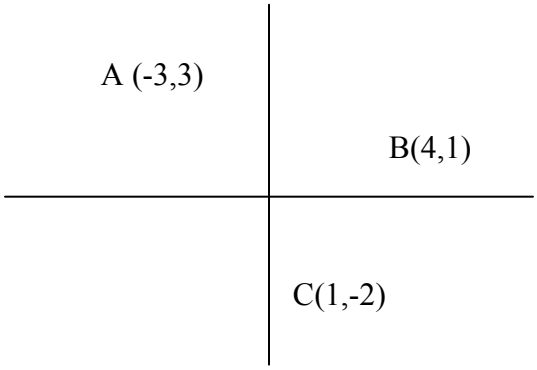
PART B: CALCULATOR OK

- Pay attention to restrictions on calculator use in individual problems!
- All answers should be fully simplified. This means all radicals in simplest radical form (<u>not decimals</u>).
- Show all work. Full credit will not always be given for the correct answer alone.

1. The following questions involve points (A, B, and C) below:



a. What is the slope of the line through points A and B? [2 points]

b. What is the equation of the line through C perpendicular to the line through A and B? [3 points]

c. What is the equation of the horizontal line through C? [2 points]

2. Inequalities

a. Solve the following inequality **algebraically** $(x-3)(2x+5) \le 0$ [4 points]

b. Based on your answer to part *a* above, sketch the solution on the number line. [2 points]

3. Simplify each expression below. All fractions should be fully reduced. Terms with the same base should be combined when possible, and all radicals should be in simplest radical form. Your answers should contain no negative exponents. [3 points each for parts a - d, 4 points for part e]

a. $\dfrac{(-2x)^2}{2x^{-3}}$

b. $\left(-\dfrac{2\sqrt{3x}}{3y^2}\right)^2$

c. $\dfrac{2}{x} - \dfrac{x-1}{x+3}$

d. $(2-3i)^2$ [write in the form $a+bi$]

e. $\dfrac{x^2-25}{x^2-3x-10} \div \dfrac{3x^2+15x}{9x^2}$

4. Find all solutions to the following equations algebraically. Any radicals in your answers should be simplified and NOT converted into decimals. [4 points each for a – e, 5 points each for f - k]

a. $\frac{2x}{3} - 4 = \frac{1}{2}$

b. $2\sqrt{x+3} = 8$

c. $x^2 + 4x = 17$

d. $x^3 + 6x^2 = 16x$

e. $\log_2(x-1) + 2 = 1$

f. $4^x - 3 = 9$

g. $x^3 - 2x^2 + 9x = 18$

h. $2\log_5 x = 1 + \log_5 4$

i. $\frac{3}{x}+1=\frac{x-2}{x+1}$

j. $x-2=\sqrt{x}$

k. $|x+3|-4=-2$

5. Answer the following questions about $f(x)=2x-5$ and $g(x)=x^2-3$. [3 points each a - c]

a. What is $(f-g)(-2)$?

b. What is $g(f(2))$?

c. What is $f(g(x))$?

6. Find the inverse of the function $f(x) = 2(x-3)^3 + 5$. [4 points]

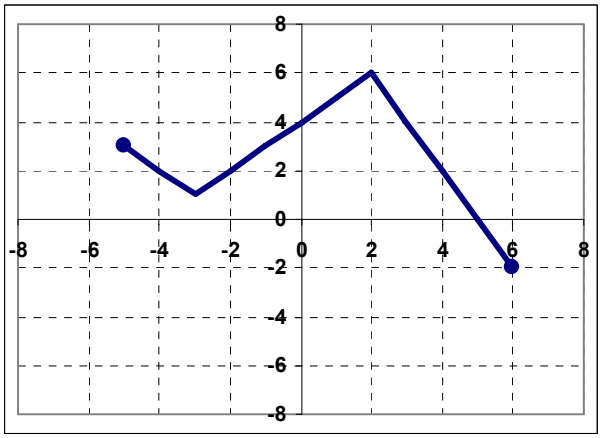
7. Use the graph of the function $f(x)$ below to answer the following questions. [2 points each]

a. What is the domain of $f(x)$?

b. What is the range of $f(x)$?

c. Solve the equation $f(x) = 4$.

d. Is the inverse of $f(x)$ a function?



On the following three applications problems (#8-10), you may use your calculator however you like. You <u>must</u> briefly describe any key steps done using your calculator; an answer without work will not receive full credit.

8. At the zoo, children's tickets cost \$4 each and adults' tickets are \$11 each. One slow morning 85 tickets were sold and a total of \$704 was collected.

a. Write an equation or system of equation describing this situation. [2 points]

b. How many of each kind of ticket was sold? [3 points]

9. A ball is launched vertically into the air from a platform 25 meters off the ground. Its height (in meters) as a function of time (in seconds) is given by the equation $h(t) = -5t^2 + 20t + 25$.

a. How long after launch does the ball hit the ground? [3 points]

b. What is the ball's maximum height and how many seconds after launch is it achieved? [4 points]

10. The first issue of Math Teacher Magazine, published in the 1930's, is currently a collector's item. It is now worth $300 and it will increase in value by 12% per year from this point forward.

a. How much will it be worth 7 years from now? [3 points]

b. How many years from now will it be worth $750? [3 points]

c. A different treasure, a 1920's Algebra 2 textbook, was worth $65 fifteen years ago. It has since tripled in value. What annual percentage increase does this represent? [3 points]