Name _____

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Lexington High School Mathematics Department

Honors Algebra 2 Final Exam 2007

SOLUTION KEY

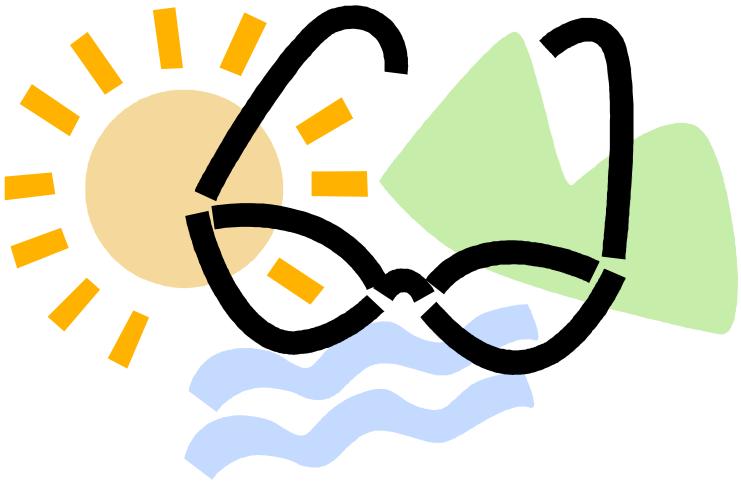
This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

Graphing calculators are permitted.

If your answer is incorrect, you may receive credit if you have shown enough correct work. Partial credit may be given.

If you use your calculator for a significant step, explain briefly what you did on the calculator.

Q 1 – 4	Q 5 - 7	Q 8 - 10	Q 11 - 12	Q 13 - 14	Q 15 - 16	Q 17	Q 18	
/16	/12	/12	/12	/12	/12	/8	/8	/92



HAVE A WONDEFUL SUMMER BREAK!

1) Perform this matrix multiplication: $\begin{bmatrix} a & -2 & 4 \\ 1 & -b & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 7 \\ -a & a \\ 0 & b \end{bmatrix}$

$$\begin{bmatrix} 5a & 5a+4b \\ 3+ab & 7-ab+5b \end{bmatrix}$$

/4

2) Rewrite as a single log: $3\log_5(2a)+\log_5(2b)-\frac{1}{2}\log_5 16$.

$$\log_5 4a^3b$$

/4

3) Consider all of the possible numbers that could be used as 5-digit ZIP codes (they range from 00000 to 99999). In what percentage of these numbers are the 5 digits all different from each other?

$$\frac{10 \bullet 9 \bullet 8 \bullet 7 \bullet 6}{10^5} = 30.24\%$$

/4

4) Write a rational function with a hole at $x=-3$ AND a horizontal asymptote $y=2$ AND a vertical asymptote of $x=5$. (There is more than one correct answer.)

$$R(x) = \frac{2x(x+3)}{(x+3)(x-5)} \text{ OR}$$

$$R(x) = \frac{(x+3)}{(x+3)(x-5)} + 2$$

/4

5) Create a data set such that the mean and median are both 10, and the modes are 5 and 8.

SAMPLE ANSWER 5 5 8 8 10 11 12 15 16

/4

6) Simplify the following expression. Your answer should have no radical signs and no negative exponents.

$$\frac{\left(2a^{-4}b^{5}\right)\sqrt[3]{b}}{\left(2ab^{2}\right)^{2}} \qquad \frac{b^{\frac{4}{3}}}{2a^{6}}$$

/4

7) Circle **all** of the matrix equations a. through f. that **could** be used to represent the system:

$$\begin{cases} x-3y=27 \\ 11x+z=4 \\ 9x-5y+7=56 \end{cases}$$

a. $\begin{bmatrix} 1 & -3 & 0 \\ 11 & 0 & 1 \\ 9 & -5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 27 \\ 4 \\ 56 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -3 & 0 \\ 11 & 0 & 1 \\ 9 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 27 \\ 4 \\ 49 \end{bmatrix}$

c. $\begin{bmatrix} 1 & -3 & 0 \\ 11 & 1 & 0 \\ 9 & -5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 27 \\ 4 \\ 56 \end{bmatrix}$

d. $\left[\begin{array}{ccc|c} 1 & -3 & 0 & x \\ 11 & 0 & 1 & y \\ 9 & -5 & 0 & z \end{array}\right]$

e. $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 27\,x \\ 11 & 1 & 0 & 4\,y \\ 9 & -5 & 0 & 56\,z \end{array}\right]$

f. $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 27 \\ 11 & 0 & 1 & 4 \\ 9 & -5 & 0 & 49 \end{array}\right]$

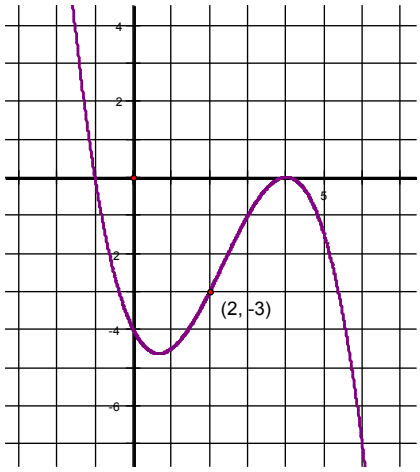
/4

8) Simplify $(x-(2-3i))(x-(2+3i))$. Write your answer in standard form.

$$x^2 - 4x + 13$$

/4

9) Write the polynomial function defined in the graph below. Write your answer in factored form.



$$f(x) = -\frac{1}{4}(x+1)(x-4)^2$$

/4

10) The polynomial function f, defined by $f(x) = 6x^3 + 29x^2 + 36x + 9$ has a zero at $x = -3$. Use this fact to completely factor the polynomial $6x^3 + 29x^2 + 36x + 9$.

$$(x+3)(2x+3)(3x+1)$$

/4

11) Solve for ***x:*** $\frac{3}{x-2}-\frac{6}{x+4}=\frac{x^2-4}{x^2+2x-8}$

$$x=-7$$

$$x=4$$

/6

12) For the function $f(x)=x^2+8x+16$ where $x>-4$, find $f^{-1}(x)$ and $(f(x))^{-1}$

$$f^{-1}(x)=\sqrt{x}-4$$

$$(f(x))^{-1}=\frac{1}{x^2+8x+16}$$

/6

If $f(x) = 2x^2 - 7$, and $g(x) = \sqrt{4x+1}$, find:

a) Find the domain and range of $f(x)$.

$$D: all\ reals$$
$$R: y \geq 7$$

b) Find the domain and range of $g(x)$.

$$D: x \geq -\frac{1}{4}$$
$$R: y \geq 0$$

c) Find $(f \circ g)(x)$.

$$(f \circ g)(x) = 8x - 5$$

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13) The first issue of Math Teacher Magazine, published in the 1930's, is currently a collector's item. It is now worth \$300 and it will increase in value by 12% per year compounded annually from this point forward.

a. How much will it be worth 7 years from now?

$$300(1.12)^7 = 663.20 \qquad \$663.20$$

b. How many years from now will it be worth \$750? Solve algebraically for full credit.

$$750 = 300(1.12)^n \qquad n = 8.09 \qquad \approx 8\ years$$

c. A different treasure, a 1920's Algebra II book, tripled in value in the last 15 years. What annual percentage increase does this represent?

$$P \bullet (1+r)^{15} = 3P$$

$(1+r)^{15} = 3$ now solve by 1/15th root or logs

$r = .076 \quad 7.6\%$

/6

14) A cube is labeled with a colored number on each face. Specifically, this is how the 6 faces are labeled:

green 1,	red 3,	purple 5,
blue 2,	red 4,	green 6.

Consider a single roll of this cube. Define these events: E = rolling an even number, G = rolling green.

Are events E and G independent? **Justify your answer using probabilities.**

YES INDEPENDENT

$P(E)\times P(G)=\frac{3}{6}\times\frac{2}{6}=\frac{6}{36}=\frac{1}{6}$

$P(E\ and\ G)=\frac{1}{6}$

OR

$P(E)=\frac{3}{6}=\frac{1}{2}$

$P(E|G)=\frac{1}{2}$

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15) The YMCA offers 12 different types of fitness classes. A member of the YMCA may choose to sign up for some, all, or none of these classes.

a. The YMCA is open each day from 9 a.m. to 9 p.m. Each of the 12 classes has to be assigned to a separate one-hour time slot. How many different ways could the schedule be made?

$$12! \quad 479{,}001{,}600$$

b. Susan is a YMCA member who wants to sign up for exactly 3 of the 12 classes. How many different ways could she make her choices?

$${}_{12}C_{3}=220$$

c. A very popular class, is the *Total Brain Math* work out. The class size is 24: 12 men and 12 women. If the YMCA has 204 members, 103 women and 101 men, how many different ways can a class be filled? Assume the class is full.

$${}_{103}C_{12}\bullet{}_{101}C_{12}=1.82282\times10^{30}$$

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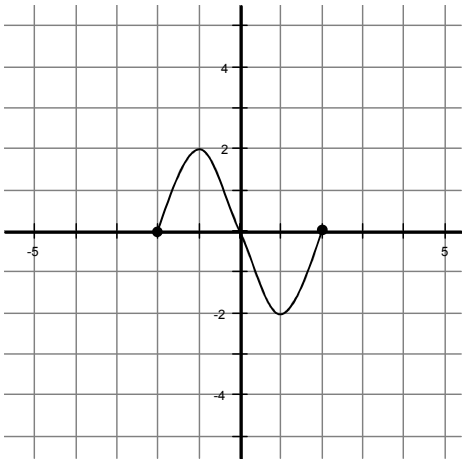
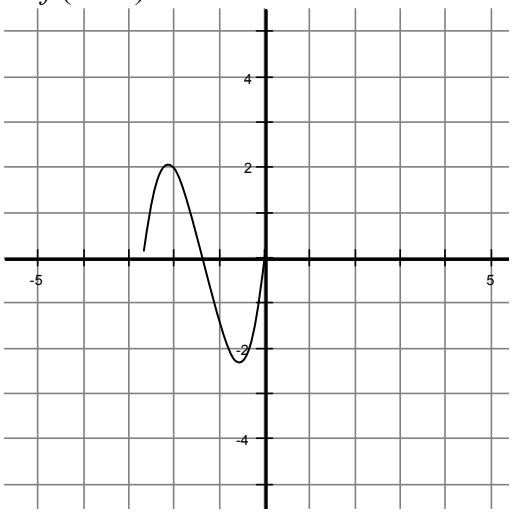
16) The graph of the function $f(x)$ is shown below.

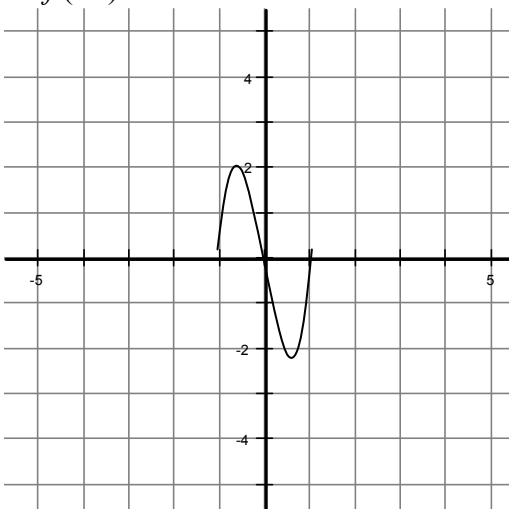


Sketch a neat and accurate graph for each of the following: NOTE: electronic sketches may be a bit OFF

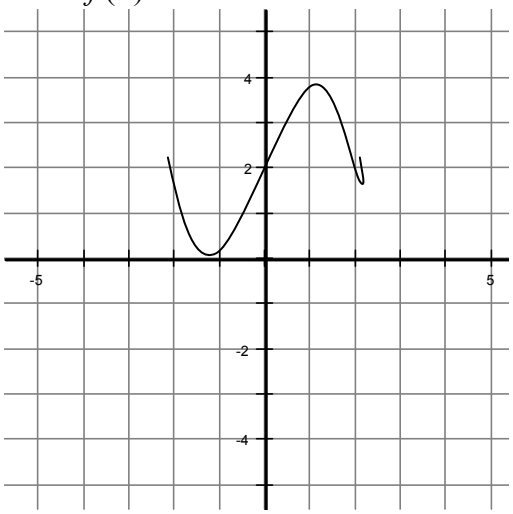
a. $f(x+2)$



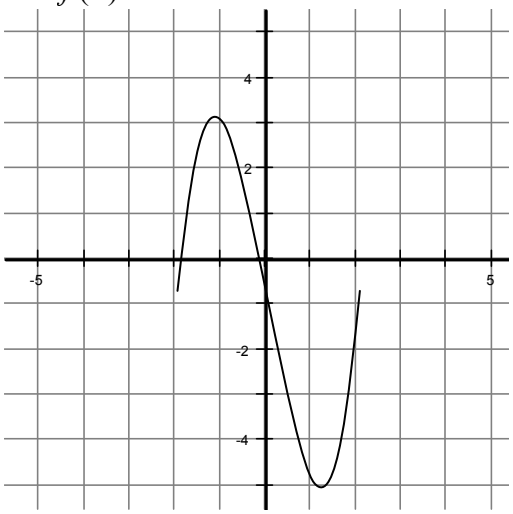
b. $f(2x)$



c. $2-f(x)$



d. $2f(x)-1$



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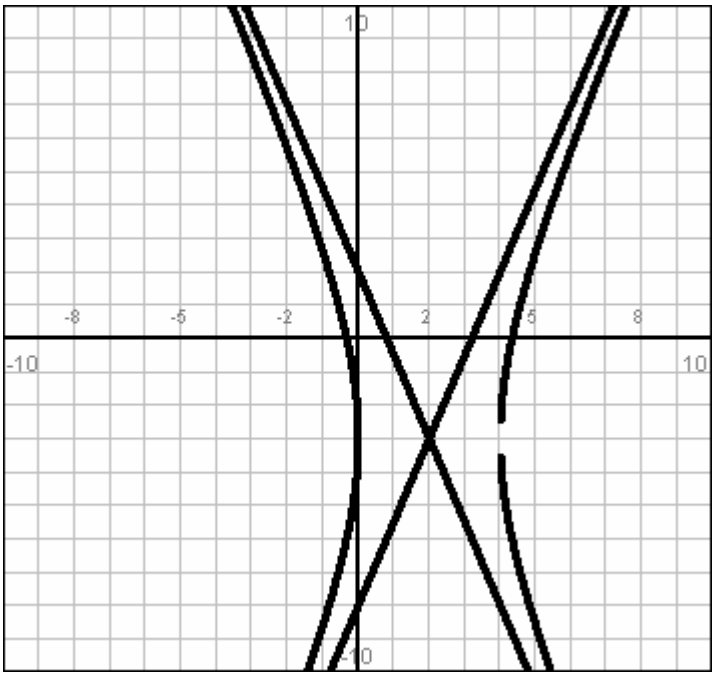
17) Given the equation $25x^2 - 4y^2 - 100x - 24y - 36 = 0$

a) Write the equation in vertex form. Work must be shown.

$$\frac{(x-2)^2}{2^2} - \frac{(y+3)^2}{5^2} = 1 \text{ or } \frac{(x-2)^2}{4} - \frac{(y+3)^2}{25} = 1$$

b) Draw the graph.



c) Write the equation of the asymptote(s) **in point-slope form**.

$$y + 3 = \pm\frac{5}{2}(x-2)$$

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