Level 1 Advanced Mathematics Final Exam
June 19, 2007

NAME: _____

Instructions

WRITE ANSWERS IN THE SPACES PROVIDED AND SHOW ALL WORK. Partial credit will not be given if work is not shown. Ask for extra paper if you need it. NOTHING ON THE EXTRA PAPER will be graded unless you explicitly write "see extra paper" on this exam paper AND you clearly indicate the problem number on the extra paper. Make sure your name is on any paper you want graded.

CALCULATORS are permitted. MAKE SURE YOU ARE IN THE CORRECT MODE ON ALL TRIG PROBLEMS. If you use your calculator for something other than numeric calculations, make sure you make clear what your inputs are.

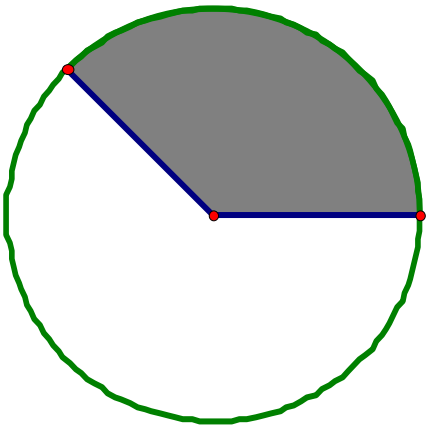
FORMULA SHEETS are attached at the end of the exam. You may detach these if you wish.

ALL PAPERS (the exam, extra paper, and the formula sheets) will be collected at the end of the exam.

POINT VALUES All problems are 2 points unless marked otherwise. There are 91 points total.

Page/max points	Score
2: 9 points	
3: 10 points	
4: 8 points	
5: 12 points	
6: 8 points	
7: 12 points	
8: 11 points	
9: 5 points	
10: 12 points	
11: 4 points	
TOTAL: 91 POINTS	

The circle at right has a radius 4. The shaded sector has an area of 6π. Write your answers in the underlined spaces. Show your work below that. ANSWERS may be in terms of π, or rounded to two decimal places. [1 point each.]



1. What fraction of a circle does the shaded sector represent?

   _____

2. What is the central angle measure of the shaded sector? (indicate the units)

   _____

3. What is the arc length of this sector?

   _____

You deposit $1,000 in an account paying 5% interest, **compounded quarterly** for t years.

4. Write a function that gives the amount in the account as a function of t. (1 point)

   _____

5. When will the balance in the account reach $1,500? Solve analytically (meaning use algebra) and round your answer to the nearest 0.1 year.

   _____

Answer the following 5 questions (numbers 6 through 10) for the function below. Write your answers in the underlined spaces. Show your work below that.

$f(x) = 2x^2 - 3$

6. Find the zero(s) of $f(x)$ if there are any.

_____

7. Is $f(x)$ even, odd, or neither? <u>Give a reason for your answer</u>. No credit for a correct answer alone.

_____

8. Find the inverse relation of $f(x)$.

_____

9. Find the domain and range of the inverse relation of $f(x)$.

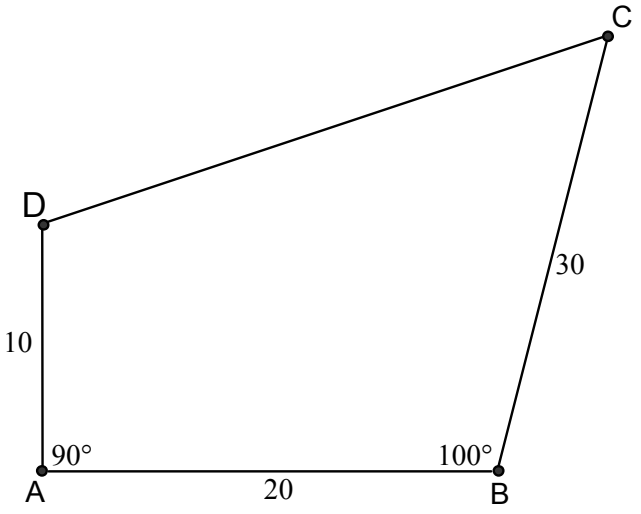
_____

10. If $g(x) = (x+4)$, find $(f \circ g)(x)$ and simplify. (Alternate notation: $(f \circ g)(x)$ is the same as $(f(g(x))$.

_____

(Use degrees for the next 4 problems.) Write your answers in the underlined spaces. Show your work below that. ANSWERS may include radicals or values to 1 decimal place.

Consider the given quadrilateral ABCD.



11. Draw segment BD and find its length.

_____

12. Determine the measure of ∠ADB.

_____

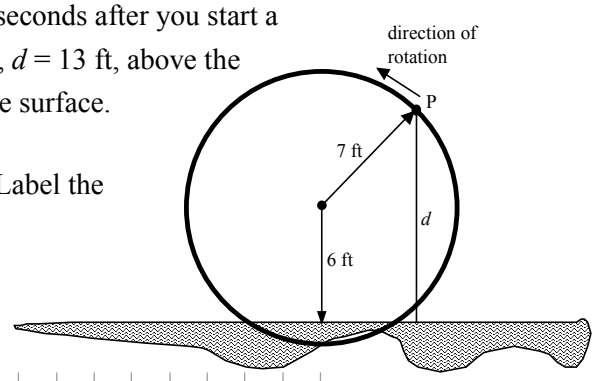
13. Find the length of segment DC.

_____

14. Find the area of quadrilateral.

_____

15. A waterwheel rotates at 6 revolutions per minute (rpm). Two seconds after you start a stopwatch, point P on the rim of the wheel is at its greatest height, $d = 13$ ft, above the surface of the water. The center of the waterwheel is 6 ft above the surface.



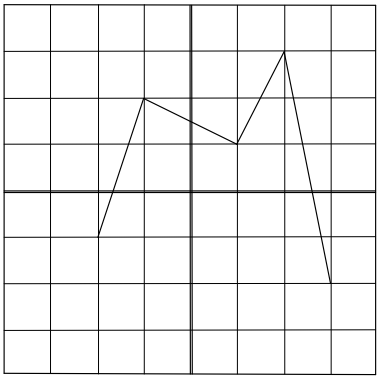
a. Sketch a graph showing P's height as a function of time. Label the coordinate of any max and min. (4 points)

b. Write an equation expressing the height of point P as a function of time. (2 pts)

c. How high above or below the water's surface will point P be at time t = 17.5? (2 pt)

d. At what time t was point P first emerging from the water? (4 pts)

The graph of $m(x)$ is shown below. Use the graph to answer each question.

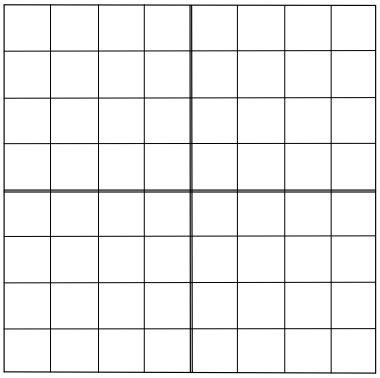


16. Is the inverse of $m(x)$ a function? Justify your response.

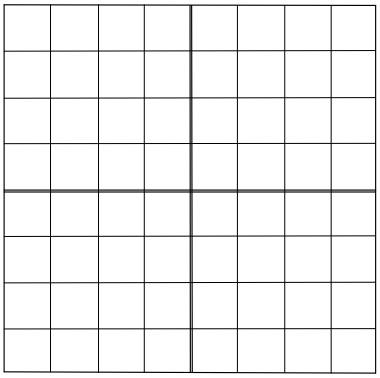
_____

(x and y scales: 1 unit per division)

17. Draw the inverse of $m(x)$ on the grid provided.



18. Graph the function $f(x) = m(x+1) - 2$ on the grid provided.



19. Write a function $g(x)$, in terms of $m(x)$, that has the graph at the right.

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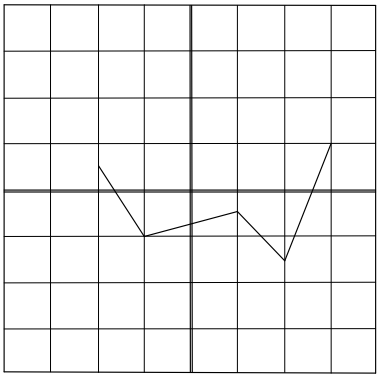


For the function $f(x)$, evaluate each limit or state that it does not exist. (1 point each)

20. $\lim_{x \to 0} f(x)$

_____

21. $\lim_{x \to -2} f(x)$

_____

22. $\lim_{x \to 0^+} f(x)$

_____

23. $\lim_{x \to 0^-} f(x)$

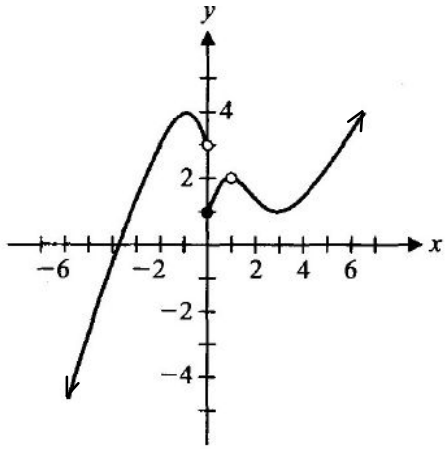
_____

24. $\lim_{x \to 1} f(x)$

_____

25. $\lim_{x \to \infty} f(x)$

_____



26. Given that $\cos 26^\circ = 0.90$ (approximately), find the following **showing work that does not require using the trig buttons on your calculator**. You will get **<u>NO CREDIT</u>** for a correct answer with no supporting work. (2 pts each)

a. Find another angle x, $0^\circ < x < 360^\circ$ where $\cos(x) = 0.9$.

b. Find an angle x, $0^\circ < x < 360^\circ$, where $\cos(x) = -0.9$.

c. Find an angle x, $0^\circ < x < 360^\circ$ where $\sin(x) = 0.9$.

Analyze the rational function by finding each of the following for: $f(x) = \dfrac{(x^2 - 1)}{(x+1)(x-5)}$

The function may or may not have all of the features below. If it does not have a particular feature, write "none" as your answer.

(1 point each)

27. Give the equations of any vertical asymptotes	28. domain
29. holes (coordinates)	30. Give the equation of the horizontal asymptote

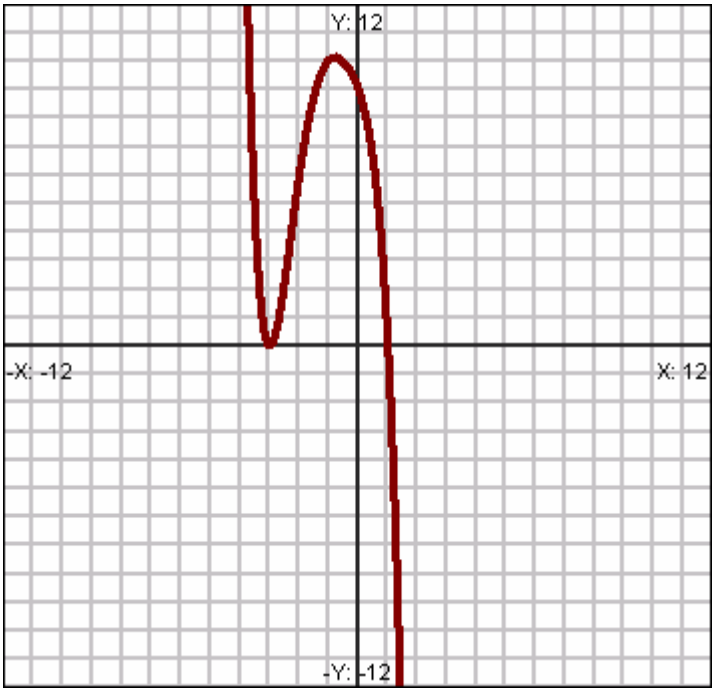
31.Solve the following logarithmic equation for x. (4 points)

$$\frac{1}{2}\log_3(x-1) = 2$$

32. Show that $\dfrac{\sin x \cos x}{1 - \cos^2 x} = \cot x$ (3 points)

The graph and some additional information about a polynomial function P(x) are represented below. Use both the graph and the information to answer the questions that follow.

- P(x) is a degree 5 polynomial with real coefficients and imaginary and real zeros.
- In the complex number system P(0 + 2i) = 0
- The graph of P(x) for real numbers, x, is given at right. The only x-intercepts are x = -3 and x = 1
- P(-2) = 6



33. Use the graph to find the real zeros (roots) of P(x) <u>and</u> the multiplicity of each zero. (Write the multiplicity <u>directly below</u> each zero on these lines). (3 points)

Zeros: _____

Multiplicities: _____

34. Given that P (0 + 2i) = 0, give another root other than those you found in the previous question (1pt)

_____

35. Given that P (-2) = 6, which of the following functions is P(x)? _____ (1 pt)

a. $\frac{1}{4}(x+3)^2(x-1)(x^2+4)$

b. $-\frac{1}{4}(x+3)^2(x-1)(x^2+4)$

c. $-\frac{1}{4}(x+3)(x-1)(x^2+4)$

d. $-4(x-3)^2(x+1)(x+2)(x-2)$

e $-4(x+3)^2(x-1)(x^2+4)$

36. Find all solutions, $0 \le x < 2\pi$, to the following equation. (Use radians) (ANSWERS may be in terms of π or decimals to 3 places) (4 points)

$2\cos x \sin x + \sin x = 0$ _____

37. How many possible arrangements are there in the letters of TAYLORHICKS? (2 point)

_____

38. How many possible 7 letter arrangements are there in the letters of TAYLORHICKS, if the first letter must be Y and the last letter a vowel? (2 point)

_____

Ms. Meegan keeps a box of crayons for the children in her class.

There are 60 crayons in the box: 10 red, 5 yellow, 8 blue, 4 orange, 12 green, 8 purple, 3 black, 6 pink, and 4 brown. 5 children take crayons from the box. Dagfin reaches in and takes a pink crayon, Marc takes a red crayon, Manasa takes a brown crayon, James takes a red crayon, Ella takes a blue crayon.

Maeve reaches in and takes two of the remaining crayons. (ANSWERS AS FRACTIONS IN LEAST TERMS OR DECIMALS TO 4 PLACES)

39. What is the probability that the first crayon Maeve grabs will be orange or green? (2 points)

_____

40. What is the probability that Maeve takes a yellow and then a pink crayon? (2 points)

_____

The average price of an adult ticket to the movie in the United States has increased, as the entries in the table below show.

x = year	y = price
1948	$0.40
1959	$0.51
1967	$1.20
1978	$2.34
1988	$4.11
1998	$4.69

Data collected from: http://www.boxofficemojo.com/about/adjuster.htm

41. Use regression on your graphing calculator to find an **exponential** model for the price data. Let 1948 = year 0 in your model. Write your answer in the underlined space below. (All constants in the model to 4 decimal places) (3 points)

_____

42. Use the model you found in the previous question to predict the price of a movie ticket in the year 2018. (ANSWER to nearest $0.01) (1 point)

_____

Level 1 Advanced Math 2007 Final Exam Reference

Compound Interest, Exponential Growth

Yearly Compound Interest, Exponential Growth $A = P(1+r)^t$	**Interest Compounded *n* times a year** $A = P\left(1+\frac{r}{n}\right)^{nt}$

Trigonometric Identities

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ $\cot(x) = \frac{\cos(x)}{\sin(x)}$	$\csc(x) = \frac{1}{\sin(x)}$ $\sec(x) = \frac{1}{\cos(x)}$ $\cot(x) = \frac{1}{\tan(x)}$	$\sin^2(x) + \cos^2(x) = 1$ $\tan^2(x) + 1 = \sec^2(x)$ $1 + \cot^2(x) = \csc^2(x)$

Laws of Sines, Cosines *Area of a Triangle*

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$	$c^2 = a^2 + b^2 - 2ab\cos(C)$	$A = \frac{1}{2}ab\sin C$

General Sinusoidal Equations

$y = C + A\cos B(x+D)$	$y = C + A\sin B(x+D)$

Counting and Probability

Permutations of *n* things taken *r* at a time. ${}_nP_r$	Combinations of *n* things taken *r* at a time. ${}_nC_r$

The Unit Circle