Name ______________________________________

Teacher (circle): Kelly Kresser Rahman

Class block (circle): A B C D G

Honors Advanced Math
Final Exam 2004

Lexington High School
Mathematics Department

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

The exam has 3 parts. Directions for each part appear below.

In total, there are 66 points that you can earn. A letter grade scale will be set by the course faculty after the tests have been graded.

Part A. Short Problems

7 questions, 2 points each, 14 points total

You must write your answers in the answer boxes.

If your answer is correct, you will receive full credit. Showing work is not required.

If your answer is incorrect, you may receive half credit if you have shown some correct work.

A good pace on this part would be to spend around 3 minutes per problem.

Part B. Medium Problems

5 problems, 4 points each, 20 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

A good pace on this part would be to spend 5-6 minutes per problem.

Part C. Long Problems

4 problems, 8 points each, 32 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

A good pace on this part would be to spend around 10 minutes per problem.

Part A. Short Problems

7 problems, 2 points each, 14 points total

1. Construct a sinusoidal function $f(x)$ that rises from a minimum point at (4, 6) to a maximum point at (7, 11).

Answer to question 1:
$f(x) =$

2. For what values of k will the second-degree equation $2y^2 + kxy + x^2 + x - 5 = 0$ be a hyperbola?

Answer to question 2:

3. A sequence t_1, t_2, t_3, … is defined by this explicit formula: $t_n = n^2$, where $n \geq 1$.
Write a **recursive definition** for the same sequence.

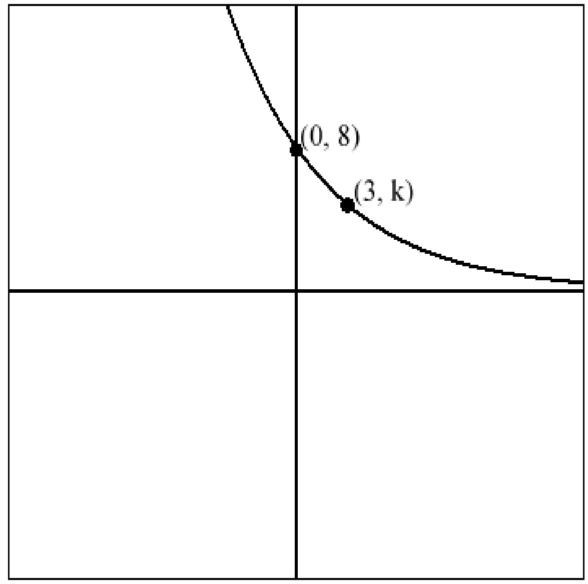
Answer to question 3:

4. The matrix equation below represents a composition of transformations that maps point (x,y) to an image point (x', y'). What are the three transformations that are composed?

$$[x\ y]\cdot\begin{bmatrix}\cos 32 & -\sin 32\\ \sin 32 & \cos 32\end{bmatrix}\cdot\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=[x'\ y']$$

Answer to question 4:
The three transformations are ____________________ followed by ____________________ followed by ____________________.

5. Write a function formula for the exponential function $F(x)$ whose graph is shown.



Answer to question 5:

$F(x) =$

6. Determine the quotient and remainder when $x^4 + x^2 - 5x + 1$ is divided by $x^2 - 5x + 1$.

Answer to question 6:

The quotient is ___________________________ and the remainder is _________________.

7. Find the solutions of the equation $x^3 - i = 0$, expressed as complex numbers in polar form.

Answer to question 7:

Part B. Medium Problems

5 problems, 4 points each, 20 points total

8. Given vectors **v** = <3, 4> and **w** = <5, –12>, compute the following.

a. Find the angle between **v** and **w**, rounded to the nearest 0.01 radian.

b. Find a *unit vector* (vector of magnitude 1) **u** that is perpendicular to **w**.

9. Consider the functions: $f(x) = \dfrac{x^2 - k^2}{x - k}$ and $g(x) = \log_k x$, where $k > 1$.

Determine whether each of the following statements is **True** or **False**. Circle **True** or **False** within each box.

True or **False** $\lim_{x \to k} f(x)$ does not exist.	**True** or **False** $\lim_{x \to k^+} f(x) = f(k)$.	**True** or **False** $\lim_{x \to -\infty} f(x) = \infty$.	**True** or **False** $f(k) = 2k$.
True or **False** $f(x)$ is continuous at $x = k$.	**True** or **False** $\lim_{x \to 0^+} g(x) = -\infty$.	**True** or **False** $\lim_{x \to \infty} g(x) = \infty$.	**True** or **False** $g(k^3) = 3k$.

10. Given the polynomial $P(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$.

a. According to the Rational Zeros (Roots) Theorem, what are the possible rational zeros of $P(x)$?

b. Find all complex zeros of $P(x)$ using analytic or symbolic methods.
Hint to get you started: Note that $P(-2i) = 0$.

11. Bart and Lisa play a game where they take turns rolling a standard 6-sided die. The first person to roll a 4 wins.

a. If Bart rolls first, what is his probability of winning the game?

b. Suppose Bart and Lisa play this game 100 times with Bart always rolling first. For each game both of them put $1 into a jackpot and the first person to roll a 4 wins the jackpot. If they both start with $100, based on expected values how much money should each of them have after the 100 games have been played.

12. Find all solutions to the equation $4 \sin 2x = -3$ in the interval $0 \le x \le 2\pi$. You may not use a graphical method, but the use of the inverse function (with results rounded to the nearest .01 radian) is acceptable.

Part C. Long Problems

4 problems, 8 points each, 32 points total

13. For this problem assume your friend has loaned you a graphing calculator (TI-83 or other similar calculator) that has certain broken features.

a. The **ln** key on the calculator is broken. How could you evaluate $\ln 4$ using only the remaining features of the calculator?

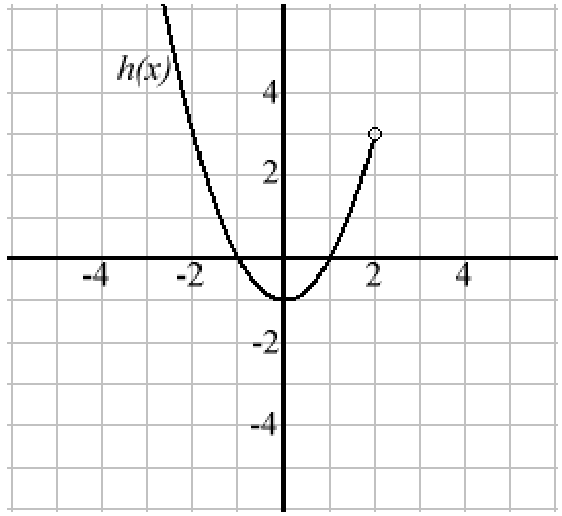
b. The calculator's ***nPr*** function does not work. How could you evaluate ${}_{15}P_6$ using a method that includes the use of the ***nCr*** function of the calculator?

c. The **cos** key on the calculator is broken. How could you evaluate $\cos(\frac{\pi}{7})$ using only the remaining features of the calculator?

d. Suppose you need to know the value of *e* accurate to five decimal places, and you don't remember it. The broken features from parts **a-c** have been fixed, but now the **e** and **e^x** operations on the calculator are broken. How could you still get the desired approximation of *e* from the calculator?

14. Consider functions $g(x)$, $h(x)$, and $j(x)$ as defined by the following equation, graph and table.

$g(x) = 1 + \sqrt{x+2}, x \geq -2$



note: $h(x)$ follows a quadratic model

x	$j(x)$
–2	4
–1	2
0	0
1	–2
2	–4

a. Find these values:

$(g \circ j \circ h)(-1) =$

$(g \circ j)(2) =$

b. Find $g^{-1}(x)$ and its domain and range. If $g^{-1}(x)$ does not exist, explain why.

c. Let $f(x) = h(x) - g(x)$. For what values of x is $f(x)$ positive?

d. On the graph of $h(x)$ above, sketch the figure that represents a reflection of $h(x)$ across the line $y = x$. Is the figure the graph of $h^{-1}(x)$? Explain why or why not.

15. A park has the shape of a quadrilateral. On a coordinate grid measured in feet, the vertices of the park are (400, 100), (0, 50), (0, 200), and (200, 250).

a. There is a walking path along each diagonal of the park. Find the lengths of these two paths.

b. The two paths along the diagonals intersect at a point inside the park. Find the measure of one of the angles formed at the intersection of the paths.

c. Find the area of the park.

16. Consider a circle, **C**, centered at (0, 0) with parametric equations $\begin{cases} x_C(t) = 3\cos(t) \\ y_C(t) = 3\sin(t) \end{cases}$.

Also consider an ellipse, **E**, centered at (2, –1) with a horizontal major axis of length 9 and a vertical minor axis of length 6.

a. Write Cartesian (rectangular) equations for **C** and **E**.

equation for **C**	equation for **E**

b. Write parametric equations for **E** of the form $\begin{cases} x_E(t) = \cdots \\ y_E(t) = \cdots \end{cases}$

c. Write a pair of equations expressing $x_E(t)$ in terms of $x_C(t)$ and $y_E(t)$ in terms of $y_C(t)$.

d. Write a single matrix equation that relates the coordinate matrices $[x_C(t) \quad y_C(t)]$ and $[x_E(t) \quad y_E(t)]$.