

Name _____

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Class block (circle): A B C D E F G H

Lexington High School Mathematics Department Honors Pre-Calculus Final Exam 2003

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

The exam has 3 parts. Directions for each part appear below.

In total, there are 72 points that you can earn. A letter grade scale will be set by the course faculty after the tests have been graded.

Part A. Short Problems

8 questions, 2 points each, 16 points total

You must write your answers in the answer boxes.

If your answer is correct, you will receive full credit. Showing work is not required.

If your answer is incorrect, you may receive half credit if you have shown some correct work.

Part B. Medium Problems

6 problems, 4 points each, 24 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

Part C. Long Problems

4 problems, 8 points each, 32 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

Part A. Short Problems

8 problems, 2 points each, 16 points total

1. Find the solution to this system of equations.

$$4A + 7B + 2C = -7$$

$$3A + 5B - 9C = 26$$

$$6A - 9B - 8C = 3$$

Answer to question 1: The solution is $(A, B, C) =$

2. Suppose $f(x) = 5\sqrt{x}$. Find $f^{-1}(x)$ and state the domain of f^{-1} .

Answer to question 2:

$$f^{-1}(x) =$$

domain of f^{-1} :

3. A linear transformation T maps the point $(0, 1)$ to $(3, 4)$ and the point $(1, 0)$ to $(-2, 5)$.
To what point does T map the point (a, b) ?

Answer to question 3: The image of (a, b) is

4. For what values of A will the series $\sum_{k=1}^{\infty} (3A)^k$ have a finite value?

Answer to question 4:

5. Re-express in as simple a form as possible:

$$\log(10m) + \log(100m) + \log(1000m)$$

Answer to question 5:

6. Assuming that $x > 0$, find an algebraic expression (not involving trigonometric functions) that is equal to $\sin(\tan^{-1}(x))$.

Answer to question 6: $\sin(\tan^{-1}(x)) =$

7. Rewrite the following equation in the standard form for a hyperbola.

$$9x^2 - 4y^2 - 18x + 16y - 11 = 0$$

Answer to question 7:

8. Find the three cube roots of 8, expressing your answers in polar form.

Answer to question 8: The cube roots of 8 are

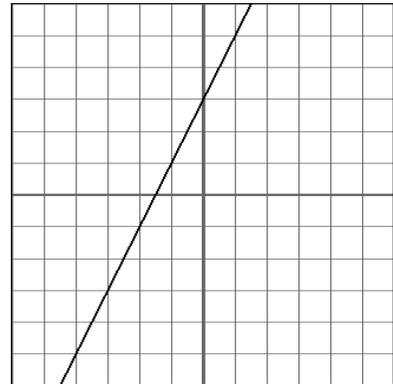
Part B. Medium Problems

6 problems, 4 points each, 24 points total

9. Using an algebraic (not graphical) method, find all solutions in the interval $0 \leq x \leq 2\pi$ to the equation $\sin(5x) = -\frac{1}{2}$. Show complete work.

10. Suppose that $\ln(f(x))$ equals the linear function whose graph is shown on the grid.

- What type of function is $f(x)$?
- Find a formula for $f(x)$.



graphing window: $[-6, 6]$ by $[-6, 6]$

11. Suppose that point $P = (2, 3)$ is rotated 120° counterclockwise around the origin. Let point Q be the image of P under this rotation. Find the coordinates of Q .

12. The following problem is about probabilities associated with a standard deck of playing cards. A standard deck contains 13 red diamonds, 13 red hearts, 13 black spades and 13 black clubs.

a. If you are dealt a hand consisting of 5 cards, find the probability of being dealt a hand containing 3 hearts and 2 black cards.

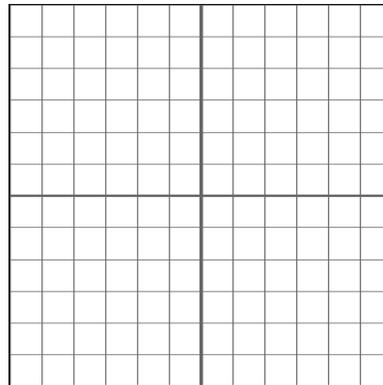
b. If you are dealt 5 cards *sequentially*, find the probability of being dealt 3 hearts *followed by* 2 black cards.

13. Suppose that $f(x) = |x|$. List the transformations needed to transform $f(x)$ into $3(f(2x)) - 1$, and sketch a graph of $3(f(2x)) - 1$.

1st trans.: _____

2nd trans.: _____

3rd trans.: _____

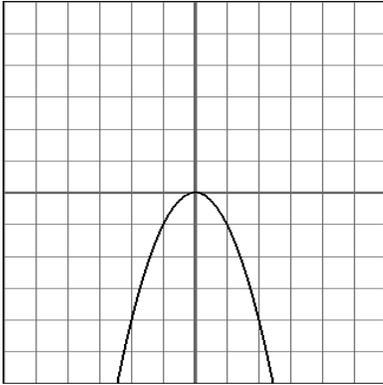


14. Given $h(x) = x^3 - x + 3$.

a. How many non-real complex zeroes does $h(x)$ have? Explain how you get your answer.

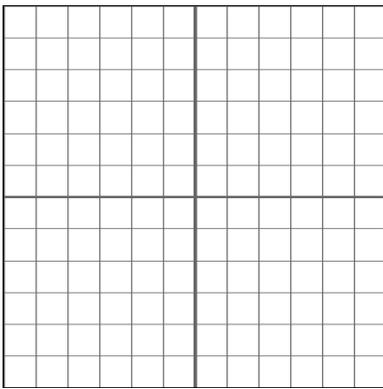
b. Prove that $h(x)$ must have an irrational zero. *Hint:* Consider the possible rational zeroes.

16. Suppose that $f(x) = |x - 1|$, and that $g(x)$ is the function whose graph is shown on the grid.



graphing window: $[-6, 6]$ by $[-6, 6]$

a. Sketch the graph of the sum function $f + g$.



b. Write a possible formula for $g(x)$.

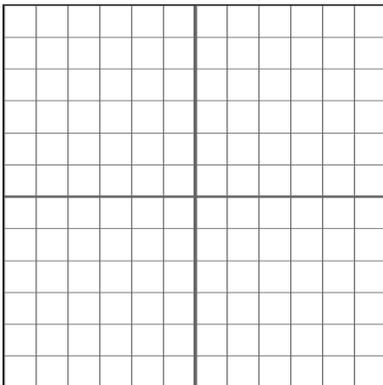
c. Based on your answer to part **b**, write a formula for the quotient function $\frac{g}{f}$, and identify the domain for this function.

function formula:

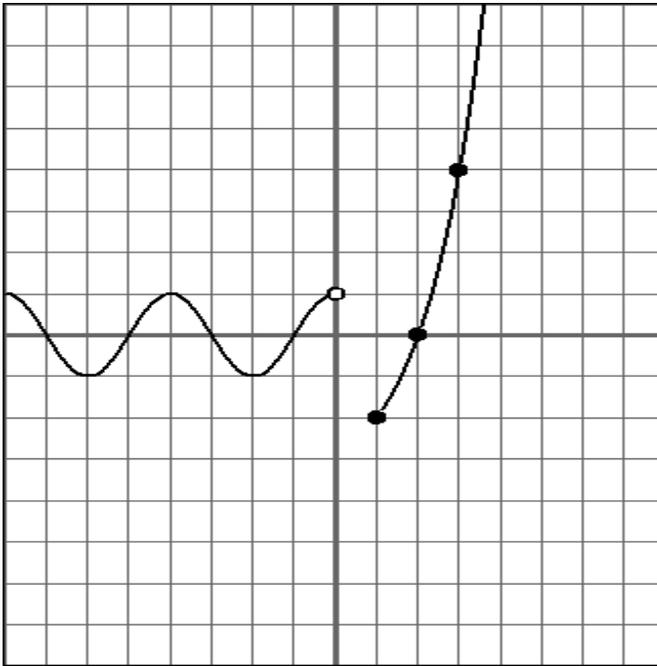
domain:

$$\frac{g}{f}(x) =$$

d. Sketch the graph of the composite function $g(f(x))$.



17. Here is a graph of a function $F(x)$, along with a table showing selected values of the function.



x	$F(x)$
-6	-1
-4	1
-2	-1
0	undef.
1	-2
2	0
3	4

graphing window: $[-8, 8]$ by $[-8, 8]$

For $-\infty < x < 0$, $F(x)$ is sinusoidal.

For $1 \leq x < \infty$, $F(x)$ has the general form $b^x + c$.

a. Identify the domain and range of F .

domain:

range:

b. Write a piecewise definition for F .

$$F(x) = \left\{ \begin{array}{l} \end{array} \right.$$

c. Identify the following limits. If a limit is not a finite number, give one of the following answers: ∞ , $-\infty$, or undefined.

$$\lim_{x \rightarrow 0^+} F(x) =$$

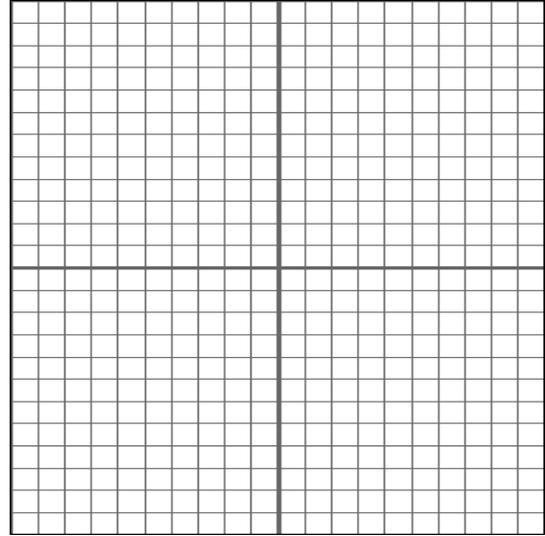
$$\lim_{x \rightarrow \infty} F(x) =$$

$$\lim_{x \rightarrow 0} F(x) =$$

$$\lim_{x \rightarrow -\infty} F(x) =$$

d. Identify the linear function on the interval $[0, 1)$ that would extend F to become a continuous function for all real numbers x .

18. Let $A = (0, 0)$. Consider the vectors
 $\vec{AB} = \langle -4, 3 \rangle$ and $\vec{AC} = \langle 5, 12 \rangle$.
 Draw them on the given grid.



a. Find the magnitudes of \vec{AB} and \vec{AC} .

b. Calculate the angle between the vectors \vec{AB} and \vec{AC} .

c. Consider triangle ABC , where vectors \vec{AB} and \vec{AC} form two of the sides.
 What is the area of $\triangle ABC$?

d. Draw the angle bisector of angle A . Find the coordinates of the point where this ray intersects side BC .