

Name \_\_\_\_\_

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# Lexington High School Mathematics Department Honors Pre-Calculus Final Exam 2002

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

The exam has 3 parts. Directions for each part appear below.

In total, there are 72 points that you can earn. A letter grade scale will be set by the course faculty after the tests have been graded.

## **Part A. Short Problems**

8 questions, 2 points each, 16 points total

You must write your answers in the answer boxes.

If your answer is correct, you will receive full credit. Showing work is not required.

If your answer is incorrect, you may receive half credit if you have shown some correct work.

## **Part B. Medium Problems**

6 problems, 4 points each, 24 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

## **Part C. Long Problems**

4 problems, 8 points each, 32 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

## Part A. Short Problems

8 problems, 4 points each, 32 points total

1. The table at the right gives the population of Massachusetts over the past several decades. Using an exponential model, predict the population of Massachusetts in the year 2015.

Year	Population
1950	4,691,000
1960	5,149,000
1970	5,689,000
1980	5,737,000
1990	6,016,000
2000	6,349,000

*Answer to question 1:* The population of Massachusetts in 2015 is predicted to be

2. A vending machine operator sells 22,000 cans of soda monthly at a price of 60 cents each. She believes that for each 5 cent increase in price, her monthly sales will drop by 2000 cans. What should her selling price be if she wishes to achieve the maximum possible revenue?

*Answer to question 2:* The selling price should be

3. Find the cube roots of the complex number  $(1 - i)$ . You may express your answer either in rectangular form or in polar form.

*Answer to question 3:* The cube roots are

4. Write parametric equations for the ray that meets the following criteria:

- the ray's terminal point is  $(3, -4)$
- the ray must pass through the point  $(-2, 5)$
- $t = 0$  must correspond to the point  $(3, -4)$

Be sure to state the domain of  $t$  values.

***Answer to question 4:***

parametric equations:

domain of  $t$  values:

5. Given  $\triangle ABC$  with  $AC = 10$ ,  $CB = 9$ , and  $\angle A = 35^\circ$ , what is the length of side  $AB$ ?

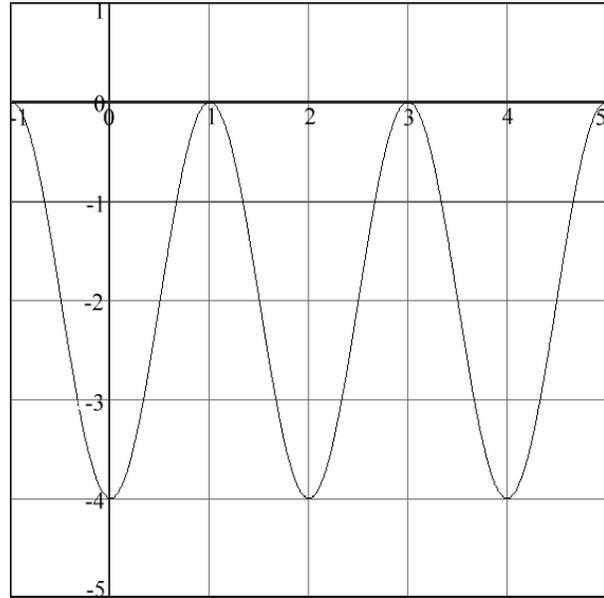
***Answer to question 5:***

6. Let  $P(x) = 6x^4 - x^3 - 6x^2 - x - 12$ .

According to the Rational Zeros Theorem, what are the ***possible*** rational zeros of  $P(x)$ ?

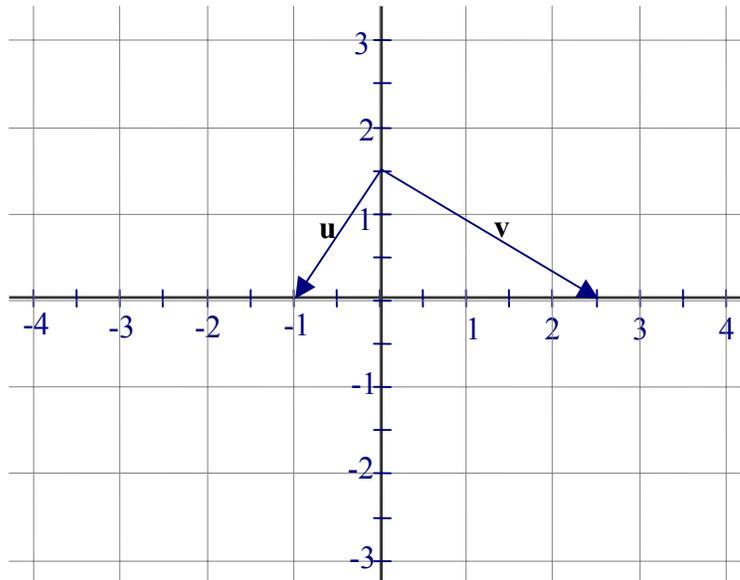
***Answer to question 6:*** The possible rational zeros are

7. Find a model involving the sine function for the given graph.



*Answer to question 7:*  $f(x) =$

8. Find the angle measure (in radians) between vectors  $\mathbf{u}$  and  $\mathbf{v}$  in the given diagram.



*Answer to question 8:* The angle measure (in radians) is

**Part B. Medium Problems**

*6 problems, 4 points each, 24 points total*

9. a. How many ways can you elect a 4-person committee from a group of 10 people?
- b. How many ways can you elect a president, a vice-president, a treasurer, and a secretary from a group of 10 people? (The jobs must be held by 4 different people.)
- c. If a 4-person committee is chosen at random from a group of 10 people, and if Amy and Katie are two of the 10 people, what is the probability that Amy and Katie are both on the committee?

10. Using your calculator, graph this equation on the grid:  $-x^2 - xy + 3y^2 - 3x + 4y = 6$ .

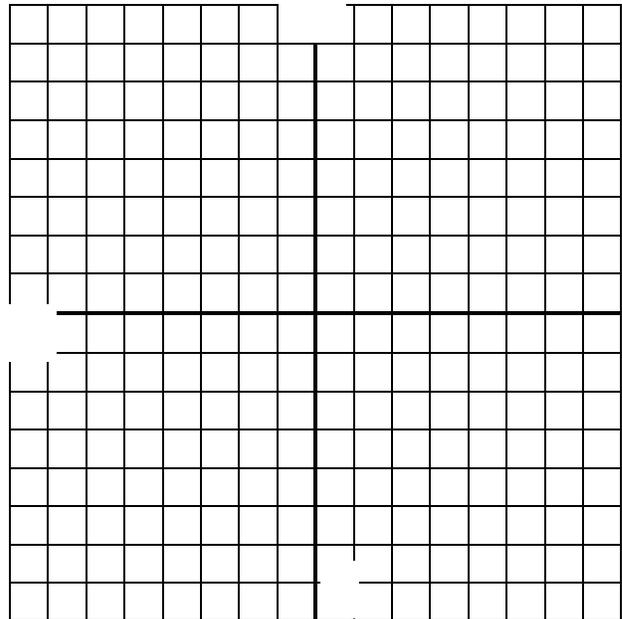
List the equation(s) you entered into the calculator

Y<sub>1</sub> =

Y<sub>2</sub> =

Y<sub>3</sub> =

Y<sub>4</sub> =



Show any additional work below.

11. Show the algebraic steps to find **all** solutions to the trigonometric equation

$$\sin^2 x - \sin x = \cos^2 x.$$

12. Answer the following questions about the series  $\sum_{n=4}^{80} (3n - 16)$ .

a. Fill in the first three terms and the last three terms of the series.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \cdots + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

b. Is the series an *arithmetic series*, a *geometric series*, or neither of these?

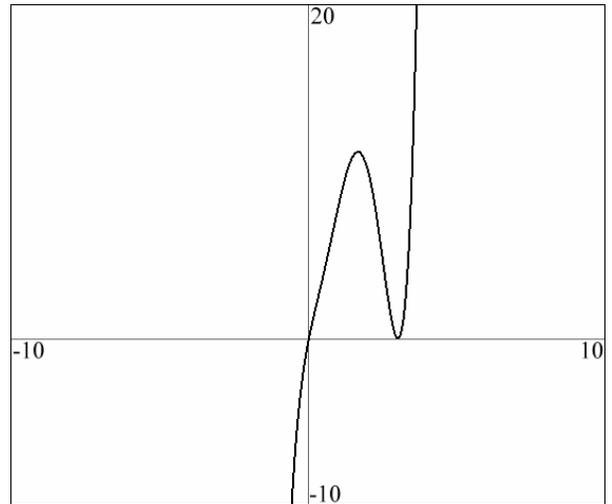
c. Find the sum of the series, or explain why the series does not have a sum.

13. You are given the following information about  $P(x)$ :

- $P(x)$  is a polynomial with real coefficients.
- The graph of  $P(x)$  for real numbers  $x$  is shown.
- The only real zeros of  $P(x)$  are 0 and 3.
- $P(i) = 0$ .

Write a possible formula for  $P(x)$ . You may leave it in either standard form or factored form

Explain how you get your formula.



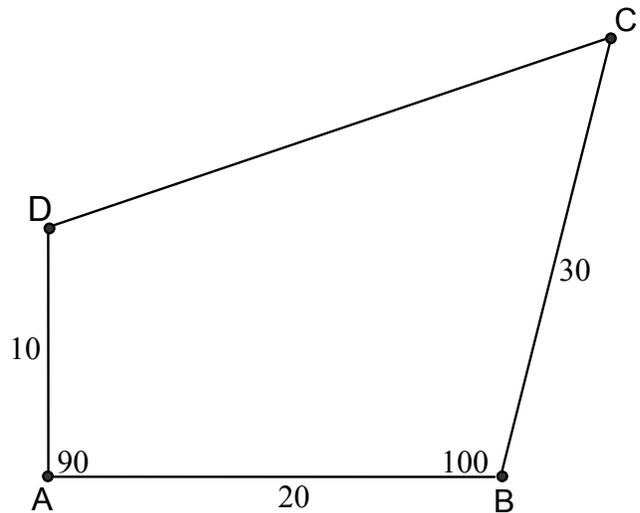
14. Find the partial fraction decomposition of  $\frac{9x^2 - 10x - 6}{x^3 - x^2 - 6x}$ .

### Part C. Long Problems

4 problems, 8 points each, 32 points total

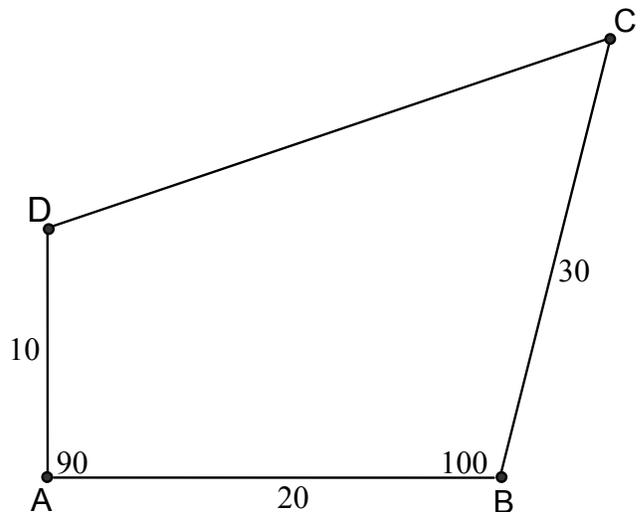
15. Consider the given quadrilateral ABCD.

- a. Find the missing side length and angle measures. Write your answers on the diagram.



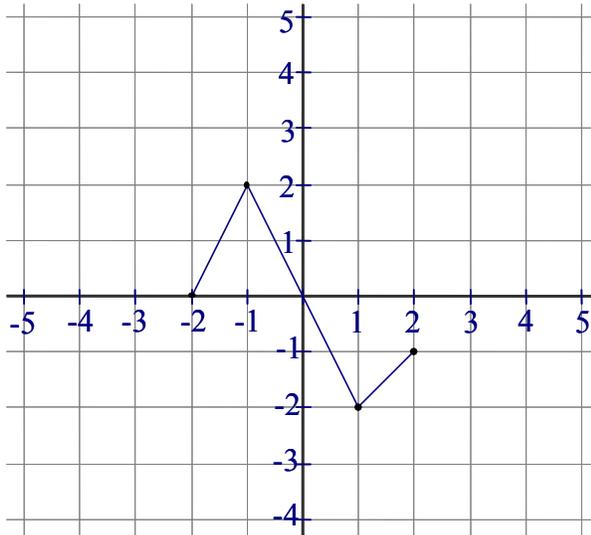
- b. Find the area of ABCD.

- c. Let point A lie at the origin of a Cartesian coordinate system with segment AB along the positive  $x$ -axis. Find the coordinates of a point P, lying on diagonal BD, such that Area of  $\triangle ABP =$  Area of  $\triangle ADP$ .



16. Each part of this problem involves transformations.

- a. Given the graph of  $F(x)$  on the first grid, sketch the graph of  $F(2x) + 1$  on the second grid.



- b. Find the periods of the functions  $\tan \theta$  and  $\tan(k\theta)$ .  
Use the unit circle definition of tangent to justify your first answer,  
and use a transformation argument to justify your second answer.
- c. A circle is described parametrically by  $x = 3 \cos t$ ,  $y = 3 \sin t$ . If this circle is stretched horizontally by a factor of 2, an ellipse is obtained. Sketch the circle and the ellipse, then write a pair of parametric equations describing the ellipse.

17. For all parts of this problem, suppose functions  $f(x)$  and  $g(x)$  are **inverses** of each other.

a. If point  $(2, 9)$  lies on the graph of  $g(x)$ , what can be said about the graph of  $f(x)$  ?

b. Re-express  $f(g(f(8)))$  in the simplest possible form.

c. Given that  $f(x) = \log_3 x$ , identify  $g(x)$ , and find  $g(-4)$ .

d. Given that  $f(x) = \log_3 x$ , re-express  $f(5a) - 4f(b) + 2$  in the form  $f(\dots)$ .

18. Do the following for the function  $f(x) = \frac{2x^2 - 6x - 20}{(x - 5)^2(x + 2)}$ .

a. Find the  $(x, y)$  coordinates of any removable discontinuities of the graph of  $f(x)$ .

b. State the domain and the range of  $f(x)$ .

domain:

range:

c. Determine each of the following limits. If the limit is not a finite number, chose one of the following answers:  $\infty, -\infty$ , or “does not exist.”

$$\lim_{x \rightarrow 5^-} f(x) =$$

$$\lim_{x \rightarrow -2} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

d. For what values of  $x$  is  $f(x)$  increasing?