Lexington High School
Mathematics Department
Honors Pre-Calculus
Final Exam 2001

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

There are three parts to the exam. Directions for each part appear below.

Part A. Multiple Choice Questions

10 questions, 4 points each, 40 points total

(Your score will be based the number of correct answers; there's no "guessing penalty.")

Do your work underneath the problems (the work will not be graded).
Write your answers (A, B, C, D, or E) in these spaces.

1. _____	**6.** _____
2. _____	**7.** _____
3. _____	**8.** _____
4. _____	**9.** _____
5. _____	**10.** _____

Part B. Written Response Questions (shorter problems)

4 questions, 5 points each, 20 points total

Show your complete work underneath the questions. Partial credit will be given.

Part C. Written Response Questions (longer or multi-part problems)

4 questions, 5 points each, 40 points total

Show your complete work underneath the questions. Partial credit will be given.

Part A. Multiple Choice Questions

10 questions, 4 points each, 40 points total

Important: Write your answers (A, B, C, D, or E) on the front page of the test.

1. Which one of these equations has ***no*** real number solutions?

(A) $x^3 + 5x^2 + 6 = 0$

(B) $x^4 + 5x^2 + 6 = 0$

(C) $x^4 - 5x^2 + 6 = 0$

(D) $x^5 + 5x^2 + 6 = 0$

(E) $x^5 + 5x^3 + 6x = 0$

2. A restaurant is building a wheelchair ramp that must rise 18 inches to meet its front door. If the ramp's angle of elevation is not allowed to exceed 6°, what is the minimum horizontal length of the ramp?

(A) less than 12 feet

(B) between 12 and 13 feet

(C) between 13 and 14 feet

(D) between 14 and 15 feet

(E) over 15 feet

3. If $F(x) = e^{\pi x}$, what is the value of $F^{-1}(e^2)$?

(A) $\pi/2$

(B) $2/\pi$

(C) π/e

(D) e/π

(E) none of these

4. What is the period of the function $g(x) = \sin\left(\frac{\pi}{1000}x\right)\cos\left(\frac{\pi}{1000}x\right)$?

(A) 0.0005

(B) 0.001

(C) 500

(D) 1000

(E) 2000

5. For any $n \geq 3$, the complex roots z of the equation $z^n = 1$ are vertices of a polygon whose perimeter is

(A) $2n$.

(B) $n \sin(2\pi/n)$.

(C) $2n \sin(2\pi/n)$.

(D) $n \sin(\pi/n)$.

(E) $2n \sin(\pi/n)$.

6. Which of these ranges of θ values is the smallest one that would produce the complete polar graph of the polar equation $r = \sin\frac{\theta}{2}$?

(A) $0 \leq \theta < \frac{\pi}{4}$

(B) $0 \leq \theta < \frac{\pi}{2}$

(C) $0 \leq \theta < \pi$

(D) $0 \leq \theta < 2\pi$

(E) $0 \leq \theta < 4\pi$

7. The limit of $Csc^{-1}(x)$ as $x \to 2$ is

(A) $\pi/6$.

(B) $\pi/3$.

(C) $\pi/2$.

(D) defined, but not one of the above values.

(E) undefined.

8. In how many different ways can the letters CALCULUSSTUDENT be arranged?

(A) 1,362,160,800

(B) 13,621,608,000

(C) 27,243,216,000

(D) 40,864,824,000

(E) 1,307,674,368,000

9. In a carnival game, the player picks a number from 1 to 6, then rolls three ordinary 6-sided dice.
If the player's number comes up all **3 times**, the player **wins $3**.
If the player's number comes up exactly **2 times**, the player **wins $2**.
If the player's number comes up just **1 time**, the player **wins $1**.
If the player's number **never** comes up, the player **loses $1**.
Which of the following statements best describes this game?

(A) The player's expected gain is about $0.04.

(B) The game is a fair game.

(C) The player's expected loss is about $0.04.

(D) The player's expected loss is about $0.08.

(E) The player's expected loss is about $0.12.

10. Suppose $Q(t)$ is a continuous function at all real numbers t. If $Q(t) = \dfrac{\sin t}{t - 2\pi}$ for all $t \neq 2\pi$, what are all the possible values for $Q(2\pi)$?

(A) It must equal 1.

(B) It can be either 1 or 2π.

(C) It must equal 2π.

(D) It can equal any real number.

(E) No values are possible.

Part B. Written Response Questions (shorter problems)

4 questions, 5 points each, 20 points total

11. What is the remainder when $x^7 - 2x^5 + 3x^3 - 4$ is divided by $x + 1$? Show the calculation you use to get your answer.

12. Solve the following equation, finding all possible values of x. Show your work.

$$\log_7 (x+1) + \log_7 (x-5) = 1$$

13. Solve the following equation, finding all possible values of x. Show your work.

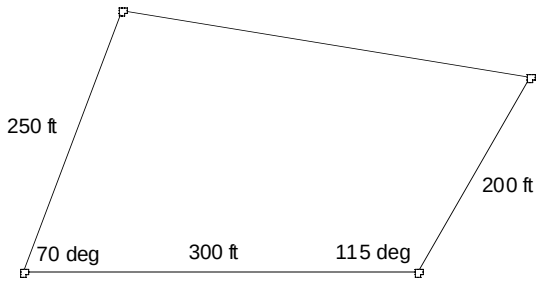
$$\tan^2 x - 4 \sec x + 5 = 0$$

14. For what values of x does the series $\sum_{j=0}^{\infty} \left(\frac{-3}{x}\right)^j$ converge? Show the work leading to your answer.

Part C. Written response (long)

4 questions, 10 points each, 40 points total

15. Calculate the area of the plot of land shown in the diagram. Show all of your work in an organized way.



16. a. Given a function f, under what circumstances will there exist an inverse function f^{-1}? For maximum credit, give both a graphical description and a non-graphical description of the necessary condition(s).

b. A function f is called an *even function* if $f(x) = f(-x)$ for all x. Does an even function *always*, *sometimes*, or *never* have an inverse function? Justify your answer.

c. Suppose f is a function that has an inverse function f^{-1}. Suppose that the domain and range of f are $1 \le x \le 5$ and $0 \le y < \infty$. What are the domain and range of f^{-1} ?

domain:

range:

d. Suppose $g(x) = \ln x$ and $h(x) = e^x$. Verify that $g(h(x)) = x$ and that $h(g(x)) = x$. What does this allow you to conclude about g and h?

17. You are given the following facts about a polynomial $P(x)$:

- The degree of $P(x)$ is less than or equal to 4.
- $P(0) = 3$.
- $P(1) = 0$.
- The graph of $P(x)$ is tangent to the x-axis at $x = 2$.
- $\lim_{x\to\infty} P(x) = -\infty$ and $\lim_{x\to-\infty} P(x) = \infty$

Determine a formula for $P(x)$, written in completely factored form.

Also, sketch a graph of $P(x)$.

18. A linear transformation T is described by the matrix equation $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

where (x, y) are the original coordinates and (x', y') are the image coordinates.

a. Find the images of the points (1, 0) and (0, 1). Draw a graph showing these two points and their images.

b. Give a geometric description of this linear transformation. Be as specific as possible.

c. ***Using a matrix calculation***, calculate the point whose image is (3, 2). Show the steps of your matrix calculation. You may use your calculator to multiply matrices, to find an inverse matrix, or for any other operation on the calculator's matrix menu.

d. Let $|\mathbf{v}|$ denote the length of vector $\mathbf{v}$. For this linear transformation T, how are $|\mathbf{v}|$ and $|T\mathbf{v}|$ related? Explain why.