Parent Involvement and Awareness:

How to Teach Your Children to Do Mental Math—Part 1

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It is important that everybody learn to do some calculations mentally when paper and pencil or a calculator is not handy. This article, Part 1, suggests a few beginnning mental math strategies that a parent might help a child learn at home.

Mental math should not be confused with the memorization of basic mathematics facts—such as knowing the times-tables by heart. While memorizing basic facts makes mental math easier, doing mathematics mentally requires both memorized facts and the manipulation (strategies) of numbers and operations in order to solve problems that are much more complex than the simple number facts we can easily memorize.

The following mental math strategies are arranged in general order from the easiest strategies children can learn to perform in their head to more difficult and challenging mental math gymnastics.

Strategies for Addition

Doing addition problems in your head is probably the best way to start doing mental math. Even young children—5, 6, and 7 year-olds—can do the easiest strategies below. While the first few may seem trivial to adults, they are a good way for children to begin learning to do mental math.

When the words "hearing" and "saying" are used in these strategies, they mean "hearing in your head" and "saying in your head."

◆ Adding One

Adding one means hearing a number, then saying one number up—or counting up one number. The best way to introduce this to your children is to say a number out loud and then, after allowing them time to think, have them tell you the next higher number. Make it fun by having your children tell you a number and then you tell them the next number. Start with low numbers and, when your children are able to count higher, move to larger numbers.

◆ Adding Two

Adding two means hearing a number, then saying the number that is two more. To do this, children can either mentally add two or count up by two. If you first teach your children to count by twos: 2, 4, 6, 8, 10, . . . etc., it will be easier for them to add two mentally. However, remember that they will also have to learn how to count by the odd numbers: 1, 3, 5, 7, 9, . . . Also, if children understand that any odd number, plus 2, will always be another odd number, and that any even number, plus two, will always be another even number, these mathematics concepts can help them check their answers mentally.

◆ Counting-On

Counting-on is one of the simple but powerful mental math strategies children can learn and is the easiest for most students—many children figure out this strategy naturally. Counting-on means a child mentally says the biggest number to add, and then counts-up the second number, one (or two) at a time. For example, in the equation 5 + 3, you start with the 5 in your head, and then count up: . . . 6, 7, 8. You might suggest to your children that if they want to add 2 + 6 in their head, they should start with the bigger number, in this case 6, and count up (. . . 7, 8) since, with addition, you can add numbers in any order and get the same answer—order does not matter. This is called the *commutative property of addition*.

When mentally counting-on, children and adults often resort to using their fingers to count up (or down), simultaneously counting on their fingers while they count in their

heads. If your children use this handy device, let them. It is not harmful if it helps to make counting-on a useful mental math strategy.

◆ Making-Ten(s)

Since ten is the basis of our number system, students who know all the single-digit combinations that equal 10 can make good use of them in doing mental math. The making-ten strategy involves memorizing the number combinations that add to ten: 7 + 3, 8 + 2, 5 + 5, etc.—they are not as useful if children need to think hard to remember these combinations. Once students memorize these, counting-on or other strategies become easier. For example, 6 + 4 = 10 may be a trivial problem, but if you know your combinations of ten, this strategy can then be extended to harder problems, such as 76 + 4, since 76 + 4 = 70 + 6 + 4 = 70 + 10 = 80—easy!

◆ Rearrange Numbers and Operations

On paper, we tend to calculate with numbers in the order they are given. Doing mathematics mentally frees us to do calculations in the order we choose and can do more easily. For example, if we do 6 – 3 + 2 + 4 + 8 in our heads, we can rearrange it as (6 + 4) + (2 + 8) – 3—two combinations of 10, then subtract 3 last. However, to do this, a child must be able to remember the numbers and rearrange them mentally. This is hard for some people.

◆ Visualizing A Mental Number Line

Number lines, such as those found on the wall in many classrooms, are a visual model of our number system and can be very helpful for children who need to see how numbers are logically arranged. If children can close their eyes and visualize a mental number line, this too can be helpful in doing mental math. The best way to help students picture a number line is to post a paper number line in your home where your children can see it and use it regularly when they do mathematics. They will begin to notice all the wonderful number patterns, the twos, the fives, the tens—and many more. If they can then see the number line when they close their eyes, they can use these patterns to do mental math.

◆ Adding Ten

The number line can teach students that adding ten is easy because ten is an easy "jump" up the number line. No matter what number you start with, the one's digit stays the same but the ten's digit increases by one. For example: 5 + 10 = 15, 12 + 10 = 22, 23 + 10 = 33, etc.

◆ Adding Nine

Once adding ten is easy to do, adding nine is the next strategy to learn. To add nine, a student just adds ten, and then counts down by one. A child would mentally say 5 + 9 = 5 + 10 – 1 = 15 – 1. Once understood, this mental math strategy is almost as simple as adding ten.

◆ Double Numbers

Making use of doubles—5 + 5, 7 + 7, etc.—is a bit harder, but can be very useful for mental math. Doubles come up often in calculations, so if all the single-digit doubles are memorized, students can combine these known facts with the mental math strategies already mentioned. For example, when faced with the problem 76 + 6, students can think of it as 70 + 6 + 6. If they remember that 6 + 6 = 12, then they can rearrange the problem as 70 + 12, and then again rearrange the problem as 70 + 10 + 2 = 82—making it an easy mental math problem.

◆ Near-Doubles

Once students have memorized their doubles, the use of near-doubles in mental math follows easily. For example, in the expression

5 + 6, if students first remember the double, 5 + 5 = 10, then it is easy to add one more, getting an answer of 11. Children actually do not have to memorize the near-doubles if they know their doubles. For example, in the equation 37 + 8, when children use the near-doubles strategy, it follows that 30 + 7 + 7 + 1 = 30 + 14 + 1 = 44 + 1 = 45.

◆ Front-end Addition

We frequently do mathematics differently in our heads than we do with paper and pencil. The typical way to add a pair of two-digit numbers is to add the digits in the ones place first, carry ten if necessary, add the digits in the tens place next, and finish by combining the tens and ones results. However, many people can keep track of these calculations more easily in their minds if they reverse this order—adding the tens first, remembering that number, then adding the ones, and only

then combining the tens and ones. For example, in the problem 65 + 26, if students first mentally calculate 60 + 20 = 80, the number 80 is pretty easy to remember—to store away mentally for a few moments. If they then add the ones, 5 + 6 = 11, they can recall the easily remembered number, and compute 80 + 11 = 91. Not everyone prefers front-end addition, but those who do often use this strategy without thinking about it.

◆ "Friendly Numbers" Strategy

Certain number pairs go together nicely and are easy to work with in our heads; we call these friendly numbers. For example, 75 + 25 totals 100—we know this well from using money. Although we do not often get many problems as simple as 75 + 25, we can combine this friendly number strategy with other mental math strategies. For example, to add 78 + 25 students would instead think 75 + 25 + 3, changing it into two friendly numbers and one easily added number instead.

◆ Balancing Strategy

Balancing numbers before you add them is a variation of the friendly number strategy. This strategy involves "borrowing" one or more from one number and "trading" it to the other number to make two numbers that are friendly. For example, 68 + 57 are not friendly numbers, but if you mentally borrow 2 from 57 and add it to the 68, the problem now becomes 70 + 55—a much easier problem to do mentally.

A Bit of Mental Math Advice

For some students these mental math strategies will be interesting and fun—and may even make them feel mathematically powerful. However, what appeals to one child may be uninteresting and hard to another. If there is one important bit of advice before you share any of these strategies with your children, it is: go slow and proceed only IF your children enjoy learning how to do mathematics in their head. A few minutes of playing with mental math are plenty—do not make it tedious. If learning mental math tricks is not fun for your children, it is best if you stop and look for other areas of mathematics, such as geometry or puzzles, that will appeal to your children more than mental math.

In Part 2 of this series, I will share mental math strategies for sraction, multiplication, division, decimals and percents.